

Institute of Mathematical Statistics and Applied Mathematics
Gheorghe Mihoc – Caius Iacob

Research Report

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Marius Rădulescu
Institute of Mathematical Statistics and Applied Mathematics, Casa Academiei Romane, Calea 13 Septembrie nr.13, RO-050711, Bucharest 5, ROMANIA,
e-mail: mradulescu.csmro@yahoo.com

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Marius Rădulescu
Institute of Mathematical Statistics and Applied Mathematics, Casa Academiei Romane, Calea 13 Septembrie nr.13, RO-050711, Bucharest 5, ROMANIA,
e-mail: mradulescu.csmro@yahoo.com

Abstract. Economic growth is frequently considered to be in conflict with sustainable development and environmental quality. With increasingly stringent environmental regulations, there is a growing need for efficient production planning models that take into account the trade-off between return and environmental costs and therefore reduce the penalties paid for overcoming the pollution levels. This research report surveys partially the current state of the literature in operations research approaches to sustainable development. Our research has been focusing in the area of crop planning and production planning under environmental constraints. Some of the models are pollution prevention models.
Keywords: multiple objective programming, production planning, environmental constraints, pollutant emission, crop planning, portfolio theory,

INTRODUCTION

Economic growth is frequently considered to be in conflict with sustainable development and environmental quality. In the last decades environmental considerations are increasingly playing an important part in our lives. Research activity directed toward environmental issues in production management has gained a great interest since the early 1990s. Various strategies have been developed to mitigate the effects of pollution. Unfortunately, these strategies are expensive to implement and may take years to have the desired effect. The pollution levels have increased very much in amount and toxicity in the last decades. People recognize that the solution to the pollution problem is pollution prevention, rather than clean up or control, although the alternative approach of source reduction may be a good starting point to the prevention process. With increasingly stringent environmental regulations, there is a growing need for efficient

production planning models that take into account the trade-off between return and environmental costs and therefore reduce the penalties paid for overcoming the pollution levels.
This research report surveys partially the current state of the literature in operations research approaches to production management in the presence of environmental constraints. A special attention is paid to the application of portfolio theory and to loss function theory to production planning models with environmental constraints. Several multiobjective programming models for industrial and agricultural production are formulated. Some of them are in continuous variables and others are in discrete variables. The two multiobjective models for the industrial production planning can be illustrated with practical examples from the textile industry.
A multiobjective model for crop planning under uncertainty is presented in the fifth paragraph of the paper. The approach is based on portfolio theory. The model takes into account weather risks, market risks and environmental risks. Input data include historical land productivity data for various crops, soil types and yield response to fertilizer/pesticide application. Several environmental levels for the application of fertilizers/pesticides, and the monetary penalties for overcoming these levels, are also considered.

1. A SURVEY OF RESEARCH IN MATHEMATICAL MODELING FOR ENVIRONMENT PROTECTION

Industrial production consists in the transformation of material resources with the participation of human beings. It is a process in which natural resources are processed or manufactured into products for consumer or re-processing purposes, in which natural resources are needed for the processing or manufacturing of products. Therefore, consumption of natural resources is a necessary condition for industrial production. Solid, liquid, gas and other wastes resulting from industrial production are bound to affect the natural environment and thus environmental changes are another inevitable result of industrial production. There is a limit to either the degree of consumption of resources, or to that of environmental change (pollution and destruction in particular) that may be tolerated. Excessive resource consumption and environmental destruction not only make industrial production unsustainable, but also destroy the basic conditions for humanity's survival.

Today the industrial development implies a colossal consumption of resources and a great impact on the environment. Therefore mankind's industrial development is challenged more severely by resource and by environmental constraints. In the 21st century, resource and environmental problems are becoming even more serious. This situation is challenging: the mankind must maintain a sustainable development and achieve its socioeconomic development goals despite of the severe resource and environmental constraints.

The concept of sustainable development supposes harmonization or simultaneous realization of the objectives connected to economic growth, environment quality and social progress. An important factor that determines the economic growth is the industrial development.

Pollution prevention is one of the most serious challenges that are currently facing the industry. At present the majority of the industrial enterprises make production plans that pollute the environment. After that they apply different methods for cleaning up the environment. The

pollution prevention seems to be the most appropriate policy the industrial enterprises must adopt. The clean up or control and the approach of source reduction may be a start to the prevention process. With increasingly stringent environmental regulations, there is a growing need for efficient production planning models that take into account the trade-off between return and environmental costs and therefore reduce the penalties paid for overcoming the pollution levels.

The importance of the problems connected with environment protection and pollution prevention represent a stimulus for the research in mathematical modeling of production processes and manufacturing systems. Production planning is a complicated task that requires cooperation among multiple functional units in any organization. In order to design an efficient production planning system, a good understanding of the environment in terms of customers, products and manufacturing processes is necessary. Although such planning exists in the company, it is often incorrectly structured due to the presence of multiple conflicting objectives. The primary difficulty in modern decision analysis is the treatment of multiple conflicting objectives. A formal decision analysis that is capable of handling multiple conflicting goals through the use of priorities may be a new frontier of management science. Application and potential use of models is the central theme of this survey. Issues for future research are presented at the end and an extensive list of publications is provided in the references at the end of the paper.

The enterprise managers become more and more aware of the potential benefits of the integrated production-planning decision support systems (DSS).

Reiborn (1999) has shown that, in the long run, the investments in systems for environment management are smaller than the benefits of the firms.

The importance of the problems connected with environment protection and pollution prevention is a stimulus for research in mathematical modeling of production processes.

There are many papers dealing with various mathematical models of production systems and related sustainable technologies (Bloemhof et al., 1995), (Chakraborty et al., 2004), (Cheng et al., 2003), (Clarke, 1987), (Daniel et al., 1997), (Dohin and Kaios, 1995), (Grauer et al., 1984), (Klassen and McLaughlin, 1999), (Klassen and Whybark, 1999), (Lemathe and Balakrishnan, 2005), (Linninger et al., 2000), (Melnyk et al., 1999), (Nijkamp and Van den Bergh, 1997), (Penkuhn et al., 1997), (Qiu et al., 2001), (Radulescu et al. 2009), (ReVelle, 2000), (Ulhoi, 1995), (Wirl, 1991), (Wirl et al. 1987), (Wu and Chang 2004, 2005). Interesting references, environmental standards and several mathematical models can be found in (Klassen and Whybark, 1999).

A rich list of references concerning papers on sustainable production technologies is given in (Radulescu et al. 2009). At present „ green manufacturing" is an objective which must be adopted by all industries with a view to reducing the environmental impact of product and production processes (Blanchard and Fabrycky 2006).

In (Melnyk et al., 1999) and (Steven and Lemathe, 1996) the authors suggest the integration of output related material and energy flows such as waste, sewage and pollutants into bills of

materials. This integration refers to the coefficients that characterize different operating procedures.

A great number of decisions about industrial production are made under uncertainty. Uncertainty governs the market price of industrial products, the price of raw materials and energy, the attempts of the firms to comply with environmental constraints etc. Mathematical modeling of the decision problems under uncertainty is much more difficult than that of the deterministic decision problems. Decision problems under uncertainty generate difficult large scale optimization problems. Often these problems are complicated by the presence of integer decision variables which are used to model the logical restrictions or cardinality restrictions. Their complexity degree increases very much when several periods are taken into account (multiperiod models). Operating in a changing and uncertain environment, firms must make strategic and operational decisions while trying to satisfy many conflicting goals. For example, in order to maximize expected profit and minimize risk, they must periodically decide when and by how much to expand capacity and even more often how much to produce, all in the face of unknown future demands, available technology, and so on. We refer to this class of problem as multi-objective decision processes under uncertainty.

The interested reader may find several decision making models for the industrial production which take into consideration the uncertainty in the following papers (Chakraborty et al., 2004), (Cheng et al., 2003), (Ierapetritou et al., 1996), (Linninger et al., 2000), (Wu and Chang 2004, 2005). An overview on the problems connected with optimization in the presence of uncertainty may be found in (Sahinidis, 2004).

2. FORMULATION OF A MULTIOBJECTIVE MODEL IN CONTINUOUS VARIABLE

In what follows a sustainable production plan means a production plan that satisfies suitable environmental constraints. We shall formulate several optimal production planning models that take into account various environmental constraints. A general multiobjective stochastic programming problem is formulated where the objective functions are the expected return of the production plan and the penalties for the case when the cumulative effect of each emission overcomes some environmental levels. The manager tries to find a production plan that maximize the expected return, minimize the pollution penalties and satisfies the environmental constraints.

Suppose that an industrial firm has the possibility to manufacture the products $P_1, P_2, \ldots, P_n$. Here by a product we understand not only the product, but the product together its production technology. Let $(\Omega, \mathcal{K}, \mathrm{P})$ be a probability space and $c_i : \Omega \times \mathbf{R}_+ \to \mathbf{R}_+$, $i = 1,2,\ldots, n$. Suppose that $c_i(\cdot, S): \Omega \to \mathbf{R}_+$ are random variables for every $i = 1,2,\ldots, n$ and $S \in \mathbf{R}_+$. If the manager invests the sum S in the manufacture of product i then he will obtain a return equal to $c_i(\cdot, S)$. A unit of the product P_i is defined as an amount of the product P_i for which the manager invested one monetary unit, say one euro. The manufacture of a product generates none, one or several pollution emissions F_1, $F_2, \ldots, F_m$ and requires p resources $R_1, R_2, \ldots, R_p$. We denote by b_{ij} the amount of pollution emission F_j per unit of product P_i and by c_{ik} the amount

of resource R_k required per unit of product P_i. Denote by r_k the maximum availability of resource R_k. Since b_{ij} depends on several random factors, including the conditions of the experiment in which the measurement of it is performed, we shall consider that b_{ij} is a random variable. Consequently we shall suppose that all $b_{ij}:\Omega \to \mathbf{R}_+$ are random variables (measurable functions).

The firm manager has a sum M of money that desires to invest in the manufacture of the products P_1, P_2,…, P_n such that to obtain a maximum expected return and a minimum risk for the pollution of the environment.

In this paper the pollution risk is measured by the penalties paid by the manager for the environment pollution.

We denote by d_{j1} the desirable or target pollution level for the pollutant emission F_j.

We denote by d_{j2} the alarm level of pollution for the pollutant emission F_j.

We denote by d_{j3} the maximum acceptable limit of pollution for the pollutant emission F_j. Of course $0 \le d_{j1} \le d_{j2} \le d_{j3}$ for every j = 1, 2,..., m. A small overcome of the level d_{j1} represents no danger for the environment. It represents only a warning that the pollution process had already began. A small overcome of the level d_{j2} represents a warning that the pollution process may have consequences for the environment.

An overcome of the level d_{j3} is a warning that the pollution process had already produced bad consequences for the environment and urgent measures must be taken in order to stop the process.

We denote by ε_{js} the admissible risk degree for overcoming the desirable risk level d_{js} s = 1, 2, 3.

Let $\mathbf{x} = (x_1, x_2, ..., x_n)$ be the fabrication plan of the manager. This means that the manager wishes to use the sum x_i in order to manufacture the product P_i, i =1,2,...,n.

If a is a real number we denote by a_+ the positive part of a, that is:

$$a_+ = \max(a,0) = \frac{|a| + a}{2}$$

We consider two approaches to the measure of the environmental risk.

A first approach is based on the assumption that the environmental penalty is proportional to the expected amount of pollutant that overcomes the pollution level. Consequently it is equal to

$$a_{js}\left(\sum_{i=1}^{n} E(b_{ij})x_i - d_{js}\right)_+$$

We denote by a_{js} the proportionality factor from the environmental penalty and by $E(b_{ij})$ the expected value of the random variable b_{ij}. In this case the overall environmental penalty is

$$\sum_{s=1}^{3}\sum_{j=1}^{m} a_{js}\left(\sum_{i=1}^{n} E(b_{ij})x_i - d_{js}\right)_+ \tag{3.1}$$

The idea of considering a desirable pollution level and environmental penalties proportional to the amount of pollutant that overcome the pollution level goes back to (Qiu et al., 2001).

In a city the value of the above level is supposed to be fixed by the city environmental protection department. It strongly depends by the degree of industrial pollution in the city. The value of this level may decrease in a polluted city and increase in a low polluted city.

A second approach is based on the assumption that the environmental penalty is proportional to the probability that the amount of pollutant overcomes the pollution levels. In this case the overall environmental penalty will be equal to

$$\sum_{s=2}^{3}\sum_{j=1}^{m} a_{js} P\left(\sum_{i=1}^{n} b_{ij}x_i \geq d_{js}\right) + \sum_{j=1}^{m} a_{j1}\left(\sum_{i=1}^{n} E(b_{ij})x_i - d_{j1}\right)_+ \tag{3.2}$$

The manager must take into account environmental constraints. We shall define two types of environmental constraints:

- Mean type environmental constraints
  These types of constraints impose some bounds on the expected amount of pollutant emissions as e.g.,

$$\sum_{i=1}^{n} E(b_{ij})x_i \leq d_{j4} \quad j = 1,2,\ldots,m \tag{3.3}$$

Here we denote by d_{j4} a number smaller or equal than d_{j3}, which is a measure of the aversion against a polluted environment. The smaller is d_{j4}, the cleaner is the environment. We shall denote by E_1 the set of all nonnegative vectors $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ that satisfy constraints (3.3).

- Safety-first type environmental constraints:
  These types of constraints impose some bounds on the probability that the pollutant emissions overcome the pollution levels as e.g.,

$$P\left(\sum_{i=1}^{n} b_{ij}x_i \geq d_{js}\right) \leq \varepsilon_{js} \qquad j = 1,2,\ldots,m \quad s = 2,3 \tag{3.4}$$

We denote by E_2 the set of all nonnegative vectors $\mathbf{x}=(x_1,x_2,...,x_n)$ that satisfy the constraints (3.4).

From the mathematical point of view no relation can be established between safety-first and mean type environmental constraints. The advantage of mean type environmental constraints is that they are easily to be checked. The drawback of the safety-first type environmental constraints is that checking them is time consuming. However the engineers that manage the production process feel that the safety-first environmental constraints capture more accurately the pollution risk than the expected value environmental constraints.

In order to use efficiently the available funds, the manager tries to find a fabrication plan $\mathbf{x}=(x_1,x_2,...,x_n)$ yielding a maximum return, minimizing the overcome of the pollution levels and allowing him to comply with environmental restrictions.

In order to find such a plan the manager must solve the following stochastic multiobjective programming problems ($q\in\{1,2\}$):

$$(\mathrm{S}_{1q})\begin{cases}\max\left[E\left[\sum_{i=1}^{n}c_i(\cdot,x_i)\right]\right]\\ \min\left(\sum_{i=1}^{n}E(b_{ij})x_i-d_{j1}\right)_+ \qquad j=1,2,...,m;\\ \min P\left(\sum_{i=1}^{n}b_{ij}x_i\geq d_{js}\right) \qquad j=1,2,...,m;\ \ s=2,3\\ \sum_{i=1}^{n}x_i=M\\ \mathbf{x}\in E_q\\ \sum_{i=1}^{n}c_{ik}x_i\leq r_k \quad k=1,2,...,p\end{cases}$$

There are several approaches allowing to reduce this problem to single objective programming problems. A common approach is to consider the objective function

$$f_1(\mathbf{x})=E\left[\sum_{i=1}^{n}c_i(\cdot,x_i)\right]-\sum_{s=2}^{3}\sum_{j=1}^{m}a_{js}P\left(\sum_{i=1}^{n}b_{ij}x_i\geq d_{js}\right)-\sum_{j=1}^{m}a_{j1}\left(\sum_{i=1}^{n}E(b_{ij})x_i-d_{j1}\right)_+$$

and to maximize $f\mathrm{BB}_{1\mathrm{BB}}$ subject to the set of feasible solutions of problem ($\mathrm{SBB}_{1p\mathrm{BB}}$). Two other approaches are presented in the next section.

3.1. THE MAXIMUM EXPECTED RETURN PROBLEM

In the maximum expected return problem the manager tries to maximize the expected return taking into account the following restrictions:

- the production plans satisfy the environmental conditions (3.3) or (3.4), that is $\mathbf{x} \in E_q$ where q=1 or q=2.
- the sum invested in the fabrication plan is equal to M.
- for all $k \in \{1,2,..., p\}$ the total amount of resource R_k required by the fabrication plan is less or equal than r_k

$$(Q_q(M)) \quad \begin{cases} \max\left[E\left[\sum_{i=1}^{n} c_i(\cdot, x_i) \right] \right] \\ \sum_{i=1}^{n} x_i = M \\ \mathbf{x} \in E_q \\ \sum_{i=1}^{n} c_{ik} x_i \le r_k \quad k = 1,2,..., p \end{cases}$$

Problem ($Q\text{BB}_{q\text{BB}}(M)$) is a stochastic parametric programming problem. The user parameter M is the sum invested by the manager for producing the fabrication plan $\mathbf{x}$.

An important problem for the manager is to find the range of parameter M.

Denote by $M\text{BB}_{1q\text{BB}}$ the optimal value of the optimization problem:

$$(\text{ABB}_{q\text{BB}}) \quad \begin{cases} \max\left[\sum_{i=1}^{n} x_i \right] \\ \mathbf{x} \in E_q \\ \sum_{i=1}^{n} c_{ik} x_i \le r_k \quad k = 1,2,..., p \end{cases}$$

In order that the set of feasible solutions of problem ($Q\text{BB}_{q\text{BB}}(M)$) be nonempty, the manager must choose the parameter M in the interval $[0, M_{1q}]$.

3.2. THE MINIMUM POLLUTION RISK PROBLEM WITH ENVIRONMENTAL CONSTRAINTS

In the minimum pollution risk problem the manager tries to minimize a linear combination (with positive coefficients) of the probabilities of overcoming the pollution levels and of the positive

part of the difference between the expected amount of pollutant emission and the desirable pollution level, taking into account the following restrictions:

- the production plans satisfy the environmental conditions (3.1) or (3.2), that is $\mathbf{x} \in E_q$ where q=1 or q=2.
- the expected return is greater than a given level W.
- the sum invested in fabrication plan is equal to M.

$$(\mathrm{P}_{1q}(M,W,\mathbf{a})) \quad \begin{cases} \min\left[\sum_{s=2}^{3}\sum_{j=1}^{m} a_{js} P\left(\sum_{i=1}^{n} b_{ij} x_i \geq d_{js}\right) + \sum_{j=1}^{m} a_{j1}\left(\sum_{i=1}^{n} E(b_{ij}) x_i - d_{j1}\right)_+\right] \\ \mathbf{x} \in E_q \\ E\left[\sum_{i=1}^{n} c_i(\cdot, x_i)\right] \geq W \\ \sum_{i=1}^{n} x_i = M \\ \sum_{i=1}^{n} c_{ik} x_i \leq r_k \quad k = 1,2,\ldots,p \end{cases}$$

The problem $\mathrm{PBB}_{1q\mathrm{BB}}(M,W,\mathbf{a})$ is a stochastic parametric programming problem. The user parameter M is the sum of money invested by the manager for producing the fabrication plan $\mathbf{x}$. The parameter $\mathbf{a}$ stands for the $m \times 3$ matrix (a_{js}). The coefficients a_{js} are positive. Their signification is connected with the penalties (charges) paid by the manager for the pollution emission. W is a lower bound for the return desired by the manager.

An important problem for the manager is to find the ranges of parameters M and W.

Denote by $M\mathrm{BB}_{1q\mathrm{BB}}$ the optimal value of the problem ($\mathrm{ABB}_{1q\mathrm{BB}}$). The range of parameter M is the interval $[0, M_{1q}]$. Suppose that the manager make a choice for the parameter M in the interval $[0, M_{1q}]$.

In order to determine a range of variation for the parameter W we shall consider the following two optimization problems:

$$(B_{1q}) \quad \begin{cases} \min\left[E\left[\sum_{i=1}^{n} c_i(\cdot, x_i)\right]\right] \\ \sum_{i=1}^{n} x_i = M \\ \mathbf{x} \in E_q \\ \sum_{i=1}^{n} c_{ik} x_i \leq r_k \quad k = 1,2,\ldots,p \end{cases}$$

and

$$(B_{2q})\begin{cases}\max\left[E\left[\sum_{i=1}^{n}c_i(\cdot,x_i)\right]\right]\\ \sum_{i=1}^{n}x_i=M\\ \mathbf{x}\in E_q\\ \sum_{i=1}^{n}c_{ik}x_i\le r_k \quad k=1,2,...,p\end{cases}$$

Denote by W_{1q} and W_{2q} the optimal values of the objective functions of problems (BBB$_{1q}$BB) and (BBB$_{2q}$BB). In order that the set of feasible solutions be nonempty, the manager must choose the parameter W in the interval $[W_{1q},W_{2q}]$.

There are several approaches in order to solve the above optimization problems. One approach require the specification of the probability distributions of the random vectors $\mathbf{b}_j=(b_{1j},b_{2j},...,b_{nj})$, j = 1, 2,..., m. The special case when the random vectors $\mathbf{b}_j$ have a multivariate normal distribution is of interest. Another approach is to treat the sample variables as an empirical distribution. The result of the optimization is valid as long as the empirical distribution accurately represents the true underlying distribution.

A common method used in to solve the problems obtained as a result of this approach is based on heuristic algorithms and computer simulation.

3.3. THE MINIMUM POLLUTION RISK PROBLEM WITH MEAN TYPE ENVIRONMENTAL CONSTRAINTS

In this section we shall study the minimum pollution risk problem when the pollution risk is defined by equation (3.1). The manager tries to minimize a linear combination (with positive coefficients) of the positive part of the difference between the expected amount of pollutant emissions and the pollution levels, taking into account the following restrictions:

- the expected amount of pollutant emission for each pollutant emission F_j is smaller than d_{j3}, the maximum acceptable limit of pollution for the pollutant emission F_j.
- the expected return is greater than a given level W.
- the sum invested in the fabrication plan is equal to M.
- for all $k\in\{1,2,...,p\}$ the total amount of resource R_k required by the fabrication plan is less or equal than r_k

$$(\mathrm{P}_2(M,W,\mathbf{a})) \quad \begin{cases} \min\left[\sum_{s=1}^{3}\sum_{j=1}^{m} a_{js}\left(\sum_{i=1}^{n} E(b_{ij})x_i - d_{js}\right)_+\right] \\ \mathbf{x} \in E_1 \\ E\left[\sum_{i=1}^{n} c_i(\cdot, x_i)\right] \geq W \\ \sum_{i=1}^{n} x_i = M \\ \sum_{i=1}^{n} c_{ik}x_i \leq r_k \qquad k = 1,2,\ldots, p \end{cases}$$

The Problem ($\mathrm{PBB}_{2\mathrm{BB}}(M,W,\mathbf{a})$) is a nonlinear deterministic parametric programming problem. The user parameter M is the sum of money invested by the manager for producing the fabrication plan $\mathbf{x}$, while parameter $\mathbf{a}$ stands for the $m \times 3$ matrix (a_{js}). The coefficients a_{js} are positive. Their meaning is connected with the penalties (charges) paid by the manager for the pollution emission. W is a lower bound for the return desired by the manager.

An important problem for the manager is to find the ranges of parameters M and W.

Denote by $M\mathrm{BB}_{2\mathrm{BB}}$ the optimal value of the problem ($\mathrm{ABB}_{2\mathrm{BB}}$). The range of parameter M is the interval $[0, M_2]$. Suppose that the manager make a choice for the parameter M in the interval $[0, M_2]$.

In order to determine a range for the parameter W we shall consider the following two optimization problems:

$$(\mathrm{C}_1) \quad \begin{cases} \min\left[E\left[\sum_{i=1}^{n} c_i(\cdot, x_i)\right]\right] \\ \sum_{i=1}^{n} x_i = M \\ \mathbf{x} \in E_1 \\ \sum_{i=1}^{n} c_{ik}x_i \leq r_k \qquad k = 1,2,\ldots, p \end{cases}$$

and

$$(C_2)\begin{cases}\max\left[E\left[\sum_{i=1}^{n} c_i(\cdot, x_i)\right]\right] \\ \sum_{i=1}^{n} x_i = M \\ \mathbf{x} \in E_1 \\ \sum_{i=1}^{n} c_{ik} x_i \le r_k \qquad k = 1,2,..., p\end{cases}$$

Denote by W_1 and W_2 the optimal values of the objective functions of problems (CBB_{1BB}) and (CBB_{2BB}). In order that the set of feasible solutions be nonempty, the manager must choose the parameter W in the interval $[W_1, W_2]$.

In the case when the profit functions are linear, that is there exist the nonnegative random variables $q_i : \Omega \to \mathbf{R}_+$ such that $c_i(\omega, s) = q_i(\omega)s$, $s > 0$, $\omega \in \Omega$, the ($PBB_{2BB}(M, W, \mathbf{a})$) model is equivalent to the linear programming model:

$$\begin{cases}\min\left[\sum_{s=1}^{3}\sum_{j=1}^{m} a_{js} y_{js}\right] \\ \sum_{i=1}^{n} E(b_{ij}) x_i - d_{js} \le y_{js} \quad j = 1,2,...,m; s = 1,2 \\ \sum_{i=1}^{n} E(b_{ij}) x_i \le d_{j4} \qquad j = 1,2,...,m; \\ \sum_{i=1}^{n} E(q_i) x_i \ge W \\ \sum_{i=1}^{n} x_i = M \\ \sum_{i=1}^{n} c_{ik} x_i \le r_k \quad k = 1,2,..., p \\ x_i \ge 0,\ y_{js} \ge 0, \quad i = 1,2,...,n;\ j = 1,2,...,m,\ s = 1,2\end{cases}$$

The decision variables in the above model are the vectors $\mathbf{x} = (x_1, x_2, ..., x_n)$ and $\mathbf{y}_s = (y_{s1}, y_{s2}, ..., y_{sm})$, s=1,2.

4. FORMULATION OF A MULTIOBJECTIVE MODEL IN DISCRETE VARIABLES

We shall formulate several discrete production planning models that take into account several environmental constraints. These models are discrete versions of the continuous variable models from the preceding paragraph. A general multi-objective programming problem is formulated in which the objective functions are the expected return of the production plan and the penalties for the case when the cumulative effect of each emission overcome some environmental levels and the financial risk of the production plan. The manager tries to find a production plan that maximize the expected return of it, minimize the pollution penalties and satisfies the environmental constraints.

Suppose that an industrial enterprise has the possibility to manufacture products of types $T_1, T_2, \ldots, T_n$. For all i=1,2,...,n, denote by c_i the selling price of a product of type T_i. Note that all c_i are random variables.

The manufacture of a product generates none, one or several pollution emissions F_1, $F_2, \ldots, F_m$ and requires p resources $R_1, R_2, \ldots, R_p$.

Denote by b_{ij} the amount of pollution emission F_j when is manufactured a product of type T_i and by c_{ik} the amount of resource R_k required for manufacturing a product of type T_i. Denote by r_k the maximum availability of resource R_k. Note that b_{ij} and c_{ik} are nonnegative numbers. The enterprise manager wants to invest a sum M of money in the range $[M_1, M_2]$ in order to manufacture products of types $T_1, T_2, \ldots, T_n$. He desires to obtain a fabrication plan $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ that gives him a maximum expected return, a minimum risk for the environment pollution and a minimum financial risk.

In the present paper the pollution risk is measured by the penalties paid by the manager for the environment pollution. Denote by d_{j1} the desirable or target pollution level for the pollutant emission F_j. Denote by d_{j2} the alarm level of pollution for the pollutant emission F_j. Denote by d_{j3} the maximum acceptable limit of pollution for the pollutant emission F_j. Of course $0 \le d_{j1} \le d_{j2} \le d_{j3}$ for every j = 1, 2,..., m. A small overcome of the level d_{j1} represent no danger for the environment. It represents only a warning that the pollution process had already began. A small overcome of the level d_{j2} represent a warning that the pollution process may have consequences for the environment. An overcome of the level d_{j3} represents a warning that the pollution process had already produced bad consequences for the environment and urgent measures must be taken in order to stop the process.

Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ be the fabrication plan of the manager. Here x_i represents the number of products of type T_i, i =1,2,...,n. Denote by p_i the production cost of a product of type T_i and by q_i a minimum quantity of products of type T_i that should be produced. Of course p_i are positive real numbers and q_i are natural numbers for all i =1,2,...,n. The production cost for the

fabrication plan $\mathbf{x} = (x_1, x_2, ..., x_n)$ is equal to $\sum_{i=1}^{n} p_i x_i$. We shall call $\mathbf{q} = (q_1, q_2, ..., q_n)$ the vector of demand. If a is a real number we shall denote by a_+ the positive part of a, that is:

$$a_+ = \max(a,0) = \frac{|a| + a}{2}$$

We shall consider that, the environmental penalty paid in the case the fabrication plan $\mathbf{x} = (x_1, x_2, ..., x_n)$ is applied is proportional to the amount of pollutant that overcomes the pollution level. Consequently in the case of pollutant emission F_j and pollution level d_{js} it is equal to

$$a_{js}\left(\sum_{i=1}^{n} b_{ij} x_i - d_{js}\right)_+$$

We denoted by a_{js} the proportionality factor from the environmental penalty. The overall environmental penalty will be in this case

$$f_1(\mathbf{x}) = \sum_{s=1}^{2}\sum_{j=1}^{m} a_{js}\left(\sum_{i=1}^{n} b_{ij} x_i - d_{js}\right)_+ \tag{4.1}$$

The idea of considering a desirable pollution level and environmental penalties proportional to the amount of pollutant that overcome the pollution level goes back to [20]. The manager must take into account environmental constraints. In our paper we shall consider constraints that impose some bounds on the expected amount of pollutant emissions:

$$\sum_{i=1}^{n} b_{ij} x_i \le d_{j4} \quad j = 1,2,..., m \tag{4.2}$$

Here we denoted by d_{j4} a number smaller or equal than d_{j3}. It measures the aversion against a polluted environment. The smaller is d_{j4}, the cleaner will be the environment. We shall denote by E_1 the set of all nonnegative vectors $\mathbf{x} = (x_1, x_2, ..., x_n)$ having integer components that satisfy: the inequalities $x_i \ge q_i$ for all i, the environmental constraints (2) and the resource constraints

$$\sum_{i=1}^{n} c_{ik} x_i \le r_k \qquad k = 1,2,..., p \tag{4.3}$$

Denote by σ_{ij} the covariance of the random variables c_i and c_j. Let $\mathbf{C} = (\sigma_{ij})$ be the covariance matrix. We shall define the financial risk of the production plan $\mathbf{x}$ as the variance of the its return $\sum_{i=1}^{n} c_i x_i$. One can easily see that

$$\mathrm{Var}\left(\sum_{i=1}^{n} c_i x_i\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j = \mathbf{x}^T \mathbf{C} \mathbf{x}$$

In order to use efficiently the sum available, the manager tries to find a fabrication plan $\mathbf{x} = (x_1, x_2, ..., x_n)$ such that it will bring a maximum return, it will minimize the overcome of the pollution levels and the financial risk and it will allow him to comply with environmental restrictions.

In order to find such a plan the manager must solve the following multiobjective programming problem:

$$(S)\begin{cases} \max\left[\sum_{i=1}^{n} (E[c_i] - p_i) x_i\right] \\ \min\left(\sum_{i=1}^{n} b_{ij} x_i - d_{js}\right)_+ \quad s = 1,2;\ j = 1,2,..., m; \\ \min\left(\sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j\right) \\ M_1 \le \sum_{i=1}^{n} p_i x_i \le M_2,\ \mathbf{x} \in E_1 \end{cases}$$

There are several approaches for reducing the above problem to single objective programming problems. Two of them are presented in the following.

4.1. A MINIMUM FINANCIAL RISK MODEL

In the minimum financial risk problems the manager tries to minimize the financial risk taking into account the following restrictions:

- the production plans satisfy the environmental and resource conditions (4.2) and (4.3), that is $\mathbf{x} \in E_1$.
- the sum M invested in the fabrication plan is in the range $[M_1, M_2]$.
- the expected return of the production plan is greater than a given value W.

The model is the following:

$$\begin{cases} \min\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j\right) \\ f_1(\mathbf{x}) \le \nu \\ \sum_{i=1}^{n}(E[c_i]-p_i)x_i \ge W \\ M_1 \le \sum_{i=1}^{n} p_i x_i \le M_2,\ \mathbf{x} \in E_1 \end{cases}$$

Here W is the parameter that controls the expected return of the production plan and ν is the parameter that controls monetarily the penalties paid for pollution.

4.2. A MAXIMUM EXPECTED RETURN MODEL

In the maximum expected return problem the manager tries to maximize the expected net return taking into account the following restrictions:

- the production plans satisfy the environmental and resource conditions (4.2) and (4.3), that is $\mathbf{x} \in E_1$.
- the sum M invested in the fabrication plan is in the range $[M_1, M_2]$.
- the financial risk is smaller than a given value τ

$$(Q)\quad \begin{cases} \max\left[\sum_{i=1}^{n}(E[c_i]-p_i)x_i - f_1(\mathbf{x})\right] \\ \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j \le \tau \\ M_1 \le \sum_{i=1}^{n} p_i x_i \le M_2,\ \mathbf{x} \in E_1 \end{cases}$$

The problem (Q) is a single objective parametric programming problem.

5 A crop planning model for sustainable agriculture

In this section a mixed integer programming model with multiple objectives for crop planning in agriculture is formulated. The model takes into account weather risks, market risks and environmental risks. Input data include historical land productivity data for various crops and soil types, yield response to fertilizer/pesticide application, unitary costs of fertilizers/pesticides, costs for crops cultivation (without using fertilizers/pesticides) on the plots.

The application of fertilizers/pesticides to agricultural crops is desirable since they contribute to the growth of agricultural production. On the other side the application of fertilizers/pesticides in great quantities, over some levels bring damages to environment and human health. In order to protect the environment and of course the people's health we shall take into account two kinds of levels: desirable levels and maximum admissible levels for the application of chemical inputs (fertilizers/pesticides). If the quantity of the chemical input is under the desirable level then no

penalty is paid by the farmer. If the quantity of the chemical input lies between the desirable level and the maximum admissible level then monetary penalties proportional to the amount that it overcomes the desirable level must be paid by the farmer.
Consider that an agricultural region is divided into several agricultural subregions.
Suppose that:

- the soil quality of a subregion is homogeneous
- the climate in a subregion is the same
- historical data on land productivities in each agricultural subregions is available for a given set of crops

For example a subregion may have non-irrigated good land quality, other region may have irrigated medium land quality etc
Consider a farm located in an agricultural region as described above, which has an agricultural land divided into several plots. Let $P_1, P_2, ..., P_m$ be the plots from the farm's land.
Suppose that each plot P_j belongs to an agricultural subregion and consequently the soil quality of each plot is homogeneous. We consider that if a plot is cultivated then it is cultivated with the same crop. Denote by S_j the area of the plot P_j j=1, 2,..., m. We consider that the farmer has to choose a crop plan from n crops $C_1, C_2, ..., C_n$, that is to make an allocation of crops to plots. In order to obtain high yields the farmer uses fertilizers/pesticides. Denote by $F_1, F_2, ..., F_k$ the set of the k fertilizers/ pesticides used by the farmer. For the fertilizer/pesticide F_r denote by q_{1r} the desirable level for the application of F_r. A crop plan that uses a quantity of fertilizer/pesticide under the desirable level is not subjected to monetary penalizations. An overcome of the desirable level is subjected to the application of environmental taxes. These taxes and the desirable and the maximum admissible levels may be established at various levels of decision making: local level, regional level, country level etc. Their amount depends on the existing level of pollution and on the community aspiration for maintaining a clean environment.
In our paper will shall consider appropriate monetary penalties for the application of fertilizers/pesticides that are proportional to the amount that exceeds the desirable level. We shall denote by q_{2ir} the maximum admissible level for the application of F_r per one hectare cultivated with crop C_i. The introduction of the pollution levels in the management of risk environment goes back to (Qiu et al 2001). For mathematical models that use the pollution levels in production planning see (Rădulescu et al 2009).
Let U_i be the cartesian product of intervals $[0, q_{2ir}]$, that is: $U_i = [0, q_{2i1}] \times [0, q_{2i2}] \times ... \times [0, q_{2ik}]$.
Consider the probability space (Ω, K, P). Denote $I = \{1,2,...,n\}, J = \{1,2,...,m\}$, $K = \{1,2,...,k\}$. For every $i \in I$, $j \in J$ we define the plot productivity functions $c_{ij} : \Omega \times U_i \to \mathbf{R}_+$ and the market price functions $b_i : \Omega \to \mathbf{R}_+$. Thus if $\mathbf{q}$ is a vector from U_i then the function $\phi_{\mathbf{q}}(\omega) = c_{ij}(\omega, \mathbf{q}), \omega \in \Omega$ is a random variable. Analogous all the functions b_i are random variables. $c_{ij}(\cdot, \mathbf{q})$ represents the quantity (in kg) of crop C_i that can be produced on one hectare of plot P_j and b_i the market price of one kg of crop C_i.
In this paper the effect of crop rotation is neglected. That is if one cultivates the same crop repeatedly on the same plot over the time, the yield decrease. The effect of crop rotation is taken into account in the papers (Alfandari et al 2009; Castellazzi et al 2008; Detlefsen and Jensen

2007; Dogliotti et al 2003; El-Nazer and McCarl 1986; Haneveld and Stegeman 2005, Mimouni et al 2000; Santos et al 2008; Vizvári et al 2009).

One can easily see that in our model the plot productivity functions do not depend on the crops cultivated previously on that plot.

Let a_{ij} be the sum of money used by the farmer in order to cultivate one hectare of plot P_j with crop C_i without using fertilizers/pesticides. For every $i \in I$, $j \in J$, $r \in K$ denote by:

- d_r - the cost of one kg of fertilizer/pesticide F_r
- w_r - the penalization coefficient for the case the quantity of fertilizer/pesticide F_r overcomes the desirable level q_{1r}.
- y_{ijr} - the decision variable representing the quantity of fertilizer/pesticide F_r applied to one hectare of plot P_j cultivated with crop C_i. Denote $\mathbf{y}_{ij} = (y_{ij1}, y_{ij2}, \ldots, y_{ijk})$.
- x_{ij} - the decision variable that takes the value 1 if the crop C_i is cultivated on plot P_j and takes value zero if the crop C_i is not cultivated on the plot P_j.
- $[M_1, M_2]$ the range for the sum of money available for investment.
- Q_i the inferior bound for the expected yield of crop C_i necessary to be obtained. It may be thought as the expected demand for crop C_i.

In our model we shall make the following assumption:

$c_{ij}(\omega, \mathbf{q}) = c_{ij0}(\omega) + \sum_{r=1}^{k} c_{ijr}(\omega) q_r$ where $\mathbf{q} = (q_1, q_2, \ldots, q_k) \in U_i$ and $\omega \in \Omega$.

The meaning of the coefficients c_{ijr} is the following:

- c_{ijr} - the additional quantity (in kg) of crop C_i obtained from one hectare of plot P_j when one kg of fertilizer/pesticide F_r is used.

The above assumption is a simplification since, in reality, the impact of an additional quantity of fertilizer F_r does depend on the levels of other fertilizers $F_{r'}$. In some cases this assumption could be restrictive as there can be synergy of fertilizers.

- c_{ij0} - the quantity of crop C_i that it is obtained from one hectare of plot P_j when no fertilizer/pesticide are used.

We shall call a crop plan a couple $(\mathbf{x}, \mathbf{y})$ where $\mathbf{x} = (x_{ij})$ is a $n \times m$ matrix with entries 0-1 and $\mathbf{y} = (\mathbf{y}_{ij})$ where $\mathbf{y}_{ij} = (y_{ij1}, y_{ij2}, \ldots, y_{ijk}) \in U_i$, $i \in I$, $j \in J$.

The environmental constraints in the model are represented by the inequalities:

$$0 \le y_{ijr} \le q_{2ir} x_{ij}, \; i \in I, \; j \in J, \; r \in K \qquad (1)$$

This means that the quantity of fertilizers/pesticides is smaller than an upper limit q_{2ir} if the plot P_j is cultivated with crop C_i. The quantity of fertilizers/pesticides is equal to zero if the plot P_j is not cultivated with crop C_i. When some crop does not need the fertilizer/pesticide F_r one can take $q_{2ir} = 0$. From the above inequality one can easily see that

$$y_{ijr} = x_{ij} y_{ijr}, \; i \in I, \; j \in J, \; r \in K \qquad (2)$$

Indeed, if x_{ij}=0 then from inequality (1) it follows that y_{ijr}=0 hence equality (2) holds. In the case x_{ij}=1 the equality (2) obviously holds.

The yield of crop C_i obtained from the plot P_j when the decisions are given by the crop plan $(\mathbf{x},\mathbf{y})$ is equal to $x_{ij}S_j\left(c_{ij0}+\sum_{r=1}^{k}c_{ijr}y_{ijr}\right)=S_j\left(c_{ij0}x_{ij}+\sum_{r=1}^{k}c_{ijr}y_{ijr}\right)$.

Take $y_{ij0}=x_{ij}$ for every $i\in I$, $j\in J$. The yield of crop C_i when the decisions are given by the crop plan $(\mathbf{x},\mathbf{y})$ is equal to $\sum_{j=1}^{m}\sum_{r=0}^{k}c_{ijr}y_{ijr}S_j$.

The sales revenue when the decisions are given by the crop plan $(\mathbf{x},\mathbf{y})$ is equal to the random variable $f_1(\mathbf{x},\mathbf{y},\cdot)=\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=0}^{k}b_ic_{ijr}S_jy_{ijr}$

The cultivation cost for the crop plan $(\mathbf{x},\mathbf{y})$ is equal to:

$$f_2(\mathbf{x},\mathbf{y})=\sum_{i=1}^{n}\sum_{j=1}^{m}\left(a_{ij}+\sum_{r=1}^{k}d_ry_{ijr}\right)S_jx_{ij}=\sum_{i=1}^{n}\sum_{j=1}^{m}\left(a_{ij}y_{ij0}+\sum_{r=1}^{k}d_ry_{ijr}\right)S_j.$$

For every real number a denote by a_+ the positive part of a that is $a_+=\max(a,0)=\frac{|a|+a}{2}$.

We define the environmental risk as the monetary penalizations paid by the farmer. It is equal to $f_3(\mathbf{x},\mathbf{y})=\sum_{r=1}^{k}w_r\left(\sum_{i=1}^{n}\sum_{j=1}^{m}y_{ijr}S_j-q_{1r}\right)_+$. One can easily see that there exists a penalization proportional to the amount of fertilizer/pesticide applied in excess over the desirable level. The environmental risk is a nonsmooth function of the crop plan $(\mathbf{x},\mathbf{y})$.

We define the gross margin (respectively the net return) obtained by the farmer when the crop plan $(\mathbf{x},\mathbf{y})$ is applied as follows: $f_4(\mathbf{x},\mathbf{y},\cdot)=f_1(\mathbf{x},\mathbf{y},\cdot)-f_2(\mathbf{x},\mathbf{y})$, (respectively)

$f_5(\mathbf{x},\mathbf{y},\cdot)=f_1(\mathbf{x},\mathbf{y},\cdot)-f_2(\mathbf{x},\mathbf{y})-f_3(\mathbf{x},\mathbf{y})$.

Note that

$$f_4(\mathbf{x},\mathbf{y},\cdot)=\sum_{i=1}^{n}\sum_{j=1}^{m}\left((b_ic_{ij0}-a_{ij})y_{ij0}+\sum_{r=1}^{k}(b_ic_{ijr}-d_r)y_{ijr}\right)S_j \text{ and}$$

$$E(f_4(\mathbf{x},\mathbf{y},\cdot))=\sum_{i=1}^{n}\sum_{j=1}^{m}\left((E(b_ic_{ij0})-a_{ij})y_{ij0}+\sum_{r=1}^{k}(E(b_ic_{ijr})-d_r)y_{ijr}\right)S_j.$$

The financial risk associated to the crop plan $(\mathbf{x},\mathbf{y})$ is defined as the variance of the gross margin or of the net return. For every $i,\alpha\in I$, $j,\beta\in J$, $r,\gamma\in K$ denote

$\rho_{ijr\alpha\beta\gamma}=E[b_ic_{ijr}b_\alpha c_{\alpha\beta\gamma}]-E[b_ic_{ijr}]\cdot E[b_\alpha c_{\alpha\beta\gamma}]$. Then the variance of the net return is equal to

$$\mathrm{Var}(f_4(\mathbf{x},\mathbf{y},\cdot))=\mathrm{Var}(f_5(\mathbf{x},\mathbf{y},\cdot))=\mathrm{Var}(f_1(\mathbf{x},\mathbf{y},\cdot))=\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=0}^{k}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{m}\sum_{\gamma=0}^{k}\rho_{ijr\alpha\beta\gamma}y_{ijr}y_{\alpha\beta\gamma}S_jS_\beta.$$

The demand constraints are represented by the inequalities:

$\sum_{j=1}^{m}\sum_{r=0}^{k}E(c_{ijr})y_{ijr}S_j\geq Q_i$, $i\in I$.

The budget constraints are represented by the inequalities: $M_1\leq f_2(\mathbf{x},\mathbf{y})\leq M_2$.

The restrictions $\sum_{i=1}^{n}x_{ij}\leq 1$, $j\in J$ show that every plot is cultivated with at most one crop.

The farmer intends to obtain optimal production plans that minimize the environmental penalties, minimize the financial risk and maximize the expected return. According to the above

requirements the multiobjective mixed integer programming model for the crop planning is the following:

$$
\begin{cases}
\min\left(\sum_{r=1}^{k} w_r \left(\sum_{i=1}^{n}\sum_{j=1}^{m} y_{ijr} S_j - q_{1r}\right)_+\right) \\
\min\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=0}^{k}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{m}\sum_{\gamma=0}^{k} \rho_{ijr\alpha\beta\gamma} y_{ijr} y_{\alpha\beta\gamma} S_j S_\beta\right) \\
\max\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\left(E(b_i c_{ij0}) - a_{ij}\right) y_{ij0} + \sum_{r=1}^{k}\left(E(b_i c_{ijr}) - d_r\right) y_{ijr}\right) S_j\right) \\
M_1 \le f_2(\mathbf{x},\mathbf{y}) \le M_2 \\
\sum_{j=1}^{m}\sum_{r=0}^{k} E(c_{ijr}) y_{ijr} S_j \ge Q_i,\ i \in I \\
0 \le y_{ijr} \le q_{2ir} y_{ij0},\ i \in I,\ j \in J,\ r \in K \\
\sum_{i=1}^{n} y_{ij0} \le 1, \text{ for every } j \in J, \\
y_{ij0} \in \{0,1\},\ i \in I,\ j \in J
\end{cases}
$$

Mathematical programming problems of the above type are very difficult to solve since they exhibit exponential complexity resulting from the presence of integer variables. Traditional approaches that apply in pure integer programming are not very helpful since the existence of continuous variables complicates their solution. Recently, simulation-based methods and heuristic algorithms have been successfully used for solving such problems. In the literature there is a small number of papers dealing with mixed integer quadratic programming problems (Lazimy 1982; Tziligakis 1999). In (Lazimy 1982) is described an approach for solving such programs based on the generalized Benders' decomposition. A new equivalent formulation that renders the program tractable is developed, under which the dual objective function is linear in the integer variables and the dual constraint set is independent of these variables. In (Tziligakis 1999) an approach based on the relaxation method is developed for solving mixed integer quadratic programs. A heuristic algorithm is built for providing tight lower and upper bounds for the mixed integer problem.

6. CONCLUSION

Currently industrial firms are facing several challenges: market competition, continuous trend of increasing pollution charges, resources conservation fees etc. The purpose of the present paper is to present various strategies based on portfolio theory and loss function theory that are intended for prevention of pollution and the exhaustion of natural resources.

The paper presents several production planning models under uncertainty that take into account both the economic and environmental problems. The models can be considered pollution prevention models. The production planning problems are multiobjective programming problems. The models presented in third, fourth and fifth paragraph are based on portfolio

theory. Here the portfolios are represented by sustainable production plans and environmental constraints are represented by several pollution levels. The fifth paragraph contains a multiobjective model for crop planning in the presence of environmental constraints. Here portfolios are represented by crop plans. In all above mentioned models ranges of the user parameters are determined. This facilitates the integration of our optimization models in environmental management decision support systems.

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