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Decomposition of Waveguides Propagating in Piezoelectric Crystals subject to Initial Fields

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The present paper deals with the study of the coupling conditions for propagation of planar guided waves in a piezoelectric semi-infinite plane (*sagittal plane*) subject to initial fields. The piezoelectric material behaves linearly and without attenuation and the waveguide propagates in a normal mode. We suppose that the material is subject to initial electro-mechanical fields. If the sagittal plane is normal to a direct, resp. inverse dyad axis, we derive that the fundamental equations' system decomposes for particular choices of the initial electric field. In this way we obtain mechanical and piezoelectric waves generalizing the classical guided waves from the case without initial fields.

1. Introduction

The problems related to electroelastic materials subject to incremental fields superposed on initial mechanical and electric fields have attracted considerable attention last period, due their complexity and to multiple applications (see papers [2, 3, 6, 20, 21, 22]). The basic equations of the theory of piezoelectric bodies subject to infinitesimal deformations and fields superposed on initial mechanical and electric fields, were established by Eringen and Maugin in their monograph [4].

In [1] the fundamental equations for piezoelectric crystals subject to initial fields have been re-established and important results concerning the dynamic and static local stability conditions of such media were obtained. In particular, the problem of plane wave propagation in piezoelectric crystals subject to initial fields was considered there. Soós and Simionescu studied in [18] the case of plane wave free propagation in 6-mm type crystals subject to initial fields. This case is important, due to its

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complexity from theoretical point of view, and its practical applications. In [8, 9] we obtained for 6-mm type crystals the plane wave velocities in closed form, analyzed the directions of polarization, defined new electro-mechanical coupling coefficients, and demonstrated the influence of initial fields on the shape of slowness surfaces. In [10] we studied the electrostrictive effect on plane wave propagation in isotropic solids subject to initial fields. In [11] we investigated the conditions of propagation of plane waves in cubic crystals subject to initial deformations and electric fields. We generalized the previous results, studying the problem of attenuated wave free propagation in an isotropic solid subject to initial electro-mechanical fields (see [12, 15]). Recent results on attenuated wave propagation along an edge, resp. on a face of a cubic crystal, subject to initial fields, are described in [13, 14, 16, 17]. Our results generalize, in a significant manner, the classical results presented in [5, 7, 19].

This paper deals with the study of the coupling conditions for propagation of planar guided waves in a piezoelectric semi-infinite plane (called *sagittal plane*). The piezoelectric material behaves linearly and without attenuation and the waveguide propagates in a normal mode. We suppose that the material is subject to initial electro-mechanical fields having small intensity. If the sagittal plane is normal to a direct, resp. inverse dyad axis (i.e. the crystal is monoclinic in the class 2, resp. in the class m), we show that the fundamental system of equations decomposes for particular choices of the initial electric field. In this way we obtain mechanical and piezoelectric waves generalizing the classical guided waves from the case without initial fields (in particular, the Bleustein-Gulyaev wave).

2. Fundamental equations. Geometric hypotheses

The basic equations of piezoelectric bodies for infinitesimal deformations and fields superposed on initial deformations and electric fields were given by Eringen and Maugin in their monograph [4]. An alternate derivation of these equations was obtained by Baesu, Fortuné and Soós in [1].

We assume the material to be an elastic dielectric, which is nonmagnetizable and conducts neither heat, nor electricity. We shall use the quasi-electrostatic approximation of the equations of balance. Furthermore, we assume that the elastic dielectric is linear and homogeneous, that the initial homogeneous deformations are infinitesimal and that the initial homogeneous electric field has small intensity. To describe this situation we use three different configurations : the *reference configuration* B_R in which at time $t = 0$ the body is undeformed and free of all fields; the *initial configuration* $\overset{\circ}{B}$ in which the body is deformed statically and carries the initial fields; the *present (current) configuration* B_t obtained from $\overset{\circ}{B}$ by applying time dependent incremental deformations and fields. In what follows, all the fields related to the initial configuration $\overset{\circ}{B}$ will be denoted by a superposed "$\circ$".

In this case the *homogeneous field equations* take the following form:

$$\overset{\circ}{\rho}\,\ddot{\boldsymbol{u}} = \operatorname{div}\boldsymbol{\Sigma},\ \operatorname{div}\boldsymbol{\Delta} = 0,$$
$$\operatorname{rot}\boldsymbol{e} = 0 \ \Leftrightarrow\ \boldsymbol{e} = -\operatorname{grad}\varphi, \tag{1}$$

where $\overset{\circ}{\rho}$ is the mass density, $\boldsymbol{u}$ is the incremental displacement from $\overset{\circ}{B}$ to B_t, $\boldsymbol{\Sigma}$ is the incremental mechanical nominal stress tensor, $\boldsymbol{\Delta}$ is the incremental electric displacement vector, $\boldsymbol{e}$ is the incremental electric field and φ is the incremental electric potential. All incremental fields involved into the above equations depend on the spatial variable $\boldsymbol{x}$ and on time t.

We suppose the following *incremental constitutive equations*:

$$\Sigma_{kl} = \overset{\circ}{\Omega}_{klmn}\, u_{m,n} + \overset{\circ}{\Lambda}_{mkl}\,\varphi_{,\,m}$$
$$\Delta_k = \overset{\circ}{\Lambda}_{kmn}\, u_{n,m} + \overset{\circ}{\epsilon}_{kl}\, e_l = \overset{\circ}{\Lambda}_{kmn}\, u_{n,m} - \overset{\circ}{\epsilon}_{kl}\,\varphi_{,\,l}. \tag{2}$$

In these equations $\overset{\circ}{\Omega}_{klmn}$ are the components of the instantaneous elasticity tensor, $\overset{\circ}{\Lambda}_{kmn}$ are the components of the instantaneous coupling tensor and $\overset{\circ}{\epsilon}_{kl}$ are the components of the instantaneous dielectric tensor. The instantaneous coefficients can be expressed in terms of the classical moduli of the material and on the initial applied fields as follows:

$$\overset{\circ}{\Omega}_{klmn} = \overset{\circ}{\Omega}_{nmlk} = c_{klmn} + \overset{\circ}{S}_{kn}\,\delta_{lm} - e_{kmn}\,\overset{\circ}{E}_l - e_{nkl}\,\overset{\circ}{E}_m - \eta_{kn}\,\overset{\circ}{E}_l\overset{\circ}{E}_m,$$
$$\overset{\circ}{\Lambda}_{mkl} = e_{mkl} + \eta_{mk}\,\overset{\circ}{E}_l,\ \ \overset{\circ}{\epsilon}_{kl} = \overset{\circ}{\epsilon}_{lk} = \delta_{kl} + \eta_{kl}, \tag{3}$$

where c_{klmn} are the components of the constant elasticity tensor, e_{kmn} are the components of the constant piezoelectric tensor, ϵ_{kl} are the components of the constant dielectric tensor, $\overset{\circ}{E}_i$ are the components of the initial applied electric field and $\overset{\circ}{S}_{kn}$ are the components of the initial applied symmetric (Cauchy) stress tensor.

From the previous field and constitutive equations we obtain the following fundamental system of equations:

$$\overset{\circ}{\rho}\,\ddot{u}_l = \overset{\circ}{\Omega}_{klmn}\, u_{m,nk} + \overset{\circ}{\Lambda}_{mkl}\,\varphi_{,mk},\qquad \overset{\circ}{\Lambda}_{kmn}\, u_{n,mk} - \overset{\circ}{\epsilon}_{kn}\,\varphi_{,nk} = 0,\ \ l = \overline{1,3}. \tag{4}$$

In what follows we shall describe the *geometric hypotheses* for our problem. The crystal is assumed to be semi-infinite, occupying the region $x_2 > 0$, and the waves are supposed to propagate along x_1 axis. The plane x_1x_2 containing the surface normal and the propagation direction is called *sagittal plane*. Furthermore, we suppose that the guide of waves has the properties invariant with time t and with x_1 variable. In these conditions, if the material behaves linearly and without attenuation, the *normal modes* will have the form:

$$u_j(\boldsymbol{x}, t) = u_j^0(x_2, x_3)\exp[\mathrm{i}(\omega t - p x_1)],\quad j = \overline{1,4}. \tag{5}$$

Here u_1, u_2, u_3 are the mechanical displacements, while u_4 stands for the electric potential φ. In the previous relations p represents the *wave number*, ω defines the *frequency* of the wave and $\mathrm{i}^2 = -1$. Using these hypotheses the equations (4) become:

$$\overset{\circ}{\Omega}_{klmn}\, u_{m,nk} + \overset{\circ}{\Lambda}_{mkl}\, \varphi_{,mk} = -\overset{\circ}{\rho}\, \omega^2 u_l, \qquad \overset{\circ}{\Lambda}_{kmn}\, u_{n,mk} = \overset{\circ}{\epsilon}_{kn}\, \varphi_{,nk}, \quad l = \overline{1,3}. \tag{6}$$

We define the non-dimensional variable $X_2 = px_2$ and we neglect the effects of diffraction in x_3 direction, so that $\partial/\partial x_3 = 0$. From the other hypotheses it yields the derivation rules $\partial/\partial x_1 = -\mathrm{i}p$ and $\partial/\partial x_2 = p\partial/\partial X_2$. Finally, we introduce the *phase velocity* of the guided wave as $V = \omega/p$.

3. The study of coupling conditions for waveguide propagation

To analyze the coupling of plane waveguide, using the previous hypotheses, we introduce the *differential operators* with complex coefficients, as follows:

$$\begin{aligned}
\overset{\circ}{\Gamma}_{il} &= \overset{\circ}{\Omega}_{i11l} - \overset{\circ}{\Omega}_{i22l}\, \frac{\partial^2}{\partial X_2^2} + \mathrm{i}(\overset{\circ}{\Omega}_{i12l} + \overset{\circ}{\Omega}_{i21l}) \frac{\partial}{\partial X_2}, \\
\overset{\circ}{\gamma}_l &= \overset{\circ}{\Lambda}_{11l} - \overset{\circ}{\Lambda}_{22l}\, \frac{\partial^2}{\partial X_2^2} + \mathrm{i}(\overset{\circ}{\Lambda}_{12l} + \overset{\circ}{\Lambda}_{21l}) \frac{\partial}{\partial X_2}, \\
\overset{\circ}{\epsilon} &= \overset{\circ}{\epsilon}_{11} - \overset{\circ}{\epsilon}_{22}\, \frac{\partial^2}{\partial X_2^2} + 2\mathrm{i}\, \overset{\circ}{\epsilon}_{12}\, \frac{\partial}{\partial X_2}.
\end{aligned} \tag{7}$$

In these conditions, after a lengthy, but elementary calculus, we obtain that the differential system (6) has the following form:

$$\begin{pmatrix}
\overset{\circ}{\Gamma}_{11} - \overset{\circ}{\rho}\, V^2 & \overset{\circ}{\Gamma}_{12} & \overset{\circ}{\Gamma}_{13} & \overset{\circ}{\gamma}_1 \\
\overset{\circ}{\Gamma}_{12} & \overset{\circ}{\Gamma}_{22} - \overset{\circ}{\rho}\, V^2 & \overset{\circ}{\Gamma}_{23} & \overset{\circ}{\gamma}_2 \\
\overset{\circ}{\Gamma}_{13} & \overset{\circ}{\Gamma}_{23} & \overset{\circ}{\Gamma}_{33} - \overset{\circ}{\rho}\, V^2 & \overset{\circ}{\gamma}_3 \\
\overset{\circ}{\gamma}_1 & \overset{\circ}{\gamma}_2 & \overset{\circ}{\gamma}_3 & -\overset{\circ}{\epsilon}
\end{pmatrix}
\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = 0. \tag{8}$$

Here the coefficients are defined by relations (7). The system (8) is a homogeneous differential system of four equations with four unknowns, i.e. the components of the mechanical displacement and the electric potential, having as coefficients complex differential operators in non-dimensional variable X_2. It generalizes the similar system from the case without initial fields, derived in [7].

In what follows we shall analyze the coupling conditions of the guided plane wave propagation in two particular cases.

3.1. Sagittal plane normal to a direct axis of order two

In this case, we suppose that *the sagittal plane* x_1x_2 *is normal to a dyad axis* (x_3 in our case). Then, the elastic constants with one index equal to 3 are zero (see

[7] for details). After a short inspection of the coefficients of the system (8), using Voigt convention, we find:

$$\overset{\circ}{\Gamma}_{13}= -[e_{15} + \mathrm{i}(e_{14} + e_{25})\frac{\partial}{\partial X_2} - e_{24}\frac{\partial^2}{\partial X_2^2}]\ \overset{\circ}{E}_1 -[\eta_{11} + 2\mathrm{i}\eta_{12}\frac{\partial}{\partial X_2} - \eta_{22}\frac{\partial^2}{\partial X_2^2}]\ \overset{\circ}{E}_1\overset{\circ}{E}_3,$$

$$\overset{\circ}{\Gamma}_{23}= -[e_{15} + \mathrm{i}(e_{14} + e_{25})\frac{\partial}{\partial X_2} - e_{24}\frac{\partial^2}{\partial X_2^2}]\ \overset{\circ}{E}_2 -[\eta_{11} + 2\mathrm{i}\eta_{12}\frac{\partial}{\partial X_2} - \eta_{22}\frac{\partial^2}{\partial X_2^2}]\ \overset{\circ}{E}_2\overset{\circ}{E}_3\ . \tag{9}$$

We can easily observe that $\overset{\circ}{\Gamma}_{13}$ and $\overset{\circ}{\Gamma}_{23}$ does not depend on the initial stress field components, but on the initial electric field components, only. Thus, $\overset{\circ}{\Gamma}_{13}=\overset{\circ}{\Gamma}_{23}= 0$ if $\overset{\circ}{E}_1=\overset{\circ}{E}_2= 0$.

Moreover, if we suppose that *the dyad axis is direct* (this means that the sagittal plane is normal to a direct axis of order two), it follows that the crystal belongs to the class 2 of the *monoclinic system* ($A_2 \parallel x_3$). In this particular case the piezoelectric constants with no index equal to 3 are zero (as described in [7]). Therefore, we obtain:

$$\overset{\circ}{\gamma}_1= (\eta_{11} + 2\mathrm{i}\eta_{12}\frac{\partial}{\partial X_2} - \eta_{22}\frac{\partial^2}{\partial X_2^2})\ \overset{\circ}{E}_1,\ \overset{\circ}{\gamma}_2= (\eta_{11} + 2\mathrm{i}\eta_{12}\frac{\partial}{\partial X_2} - \eta_{22}\frac{\partial^2}{\partial X_2^2})\ \overset{\circ}{E}_2\ . \tag{10}$$

So, we obtain that $\overset{\circ}{\gamma}_1=\overset{\circ}{\gamma}_2= 0$ if $\overset{\circ}{E}_1=\overset{\circ}{E}_2= 0$.

In conclusion, we derive the following result concerning the decomposition of the fundamental system (8).

If the axis x_3 is a direct dyad axis and if $\overset{\circ}{E}_1=\overset{\circ}{E}_2= 0$, the system (8) reduces to two independent subsystems, as follows:

- The first subsystem:

$$\begin{pmatrix} \overset{\circ}{\Gamma}_{11} - \overset{\circ}{\rho}\,V^2 & \overset{\circ}{\Gamma}_{12} \\ \overset{\circ}{\Gamma}_{12} & \overset{\circ}{\Gamma}_{22} - \overset{\circ}{\rho}\,V^2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0. \tag{11}$$

  It defines a non-piezoelectric guided wave, polarized in the sagittal plane x_1x_2, which depends on the initial stress field, only. We shall denote it by $\overset{\circ}{P}_2$. These characterizations are due to the form of the involved coefficients:

$$\overset{\circ}{\Gamma}_{11}= c_{11}+ \overset{\circ}{S}_{11} +2\mathrm{i}(c_{16}+ \overset{\circ}{S}_{12})\frac{\partial}{\partial X_2} - (c_{66}+ \overset{\circ}{S}_{22})\frac{\partial^2}{\partial X_2^2},$$

$$\overset{\circ}{\Gamma}_{12}= c_{16} + \mathrm{i}(c_{12} + c_{66})\frac{\partial}{\partial X_2} - c_{26}\frac{\partial^2}{\partial X_2^2}, \tag{12}$$

$$\overset{\circ}{\Gamma}_{22}= c_{66}+ \overset{\circ}{S}_{11} +2\mathrm{i}(c_{26}+ \overset{\circ}{S}_{12})\frac{\partial}{\partial X_2} - (c_{22}+ \overset{\circ}{S}_{22})\frac{\partial^2}{\partial X_2^2}.$$

- The second subsystem:

$$\begin{pmatrix} \overset{\circ}{\Gamma}_{33} - \overset{\circ}{\rho} V^2 & \overset{\circ}{\gamma}_3 \\ \overset{\circ}{\gamma}_3 & -\overset{\circ}{\epsilon} \end{pmatrix} \begin{pmatrix} u_3 \\ u_4 \end{pmatrix} = 0. \tag{13}$$

It has as solution a transverse-horizontal wave, with polarization after the axis x_3, which is piezoelectric and electrostrictive active, and depends on the initial mechanical and electrical fields. It is denoted by $\overset{\circ}{\overline{TH}}$ and generalizes the famous Bleustein-Gulyaev wave (see [7], to compare). The components involved into this equation have the form:

$$\begin{aligned} \overset{\circ}{\Gamma}_{33} &= c_{55} + \overset{\circ}{S}_{11} + 2\mathrm{i}(c_{45} + \overset{\circ}{S}_{12}) \frac{\partial}{\partial X_2} - (c_{44} + \overset{\circ}{S}_{22}) \frac{\partial^2}{\partial X_2^2} \\ &-2[e_{15} + \mathrm{i}(e_{14} + e_{25}) \frac{\partial}{\partial X_2} - e_{24} \frac{\partial^2}{\partial X_2^2}] \overset{\circ}{E}_3 - [\eta_{11} + 2\mathrm{i}\eta_{12} \frac{\partial}{\partial X_2} - \eta_{22} \frac{\partial^2}{\partial X_2^2}] \overset{\circ}{E}{}_3^2, \\ \overset{\circ}{\gamma}_3 &= e_{15} + \mathrm{i}(e_{14} + e_{25}) \frac{\partial}{\partial X_2} - e_{24} \frac{\partial^2}{\partial X_2^2} + [\eta_{11} + 2\mathrm{i}\eta_{12} \frac{\partial}{\partial X_2} - \eta_{22} \frac{\partial^2}{\partial X_2^2}] \overset{\circ}{E}_3, \\ \overset{\circ}{\epsilon} &= \overset{\circ}{\epsilon}_{11} + 2\mathrm{i} \overset{\circ}{\epsilon}_{12} \frac{\partial}{\partial X_2} - \overset{\circ}{\epsilon}_{22} \frac{\partial^2}{\partial X_2^2} = 1 + \eta_{11} + 2\mathrm{i}\eta_{12} \frac{\partial}{\partial X_2} - (1 + \eta_{22}) \frac{\partial^2}{\partial X_2^2}. \end{aligned} \tag{14}$$

3.2. Sagittal plane parallel to a mirror plane

We suppose now that *the sagittal plane* x_1x_2 *is normal to an inverse dyad axis* (x_3 in our case) or, equivalently, if *the sagittal plane is parallel to a mirror plane* M. It follows that the crystal belongs to the class m of the *monoclinic system* ($M \perp x_3$). In this particular case the elastic constants with one index equal to 3 are zero, as well as the piezoelectric constants with one index equal to 3, which vanish (see [7] for details).

Analyzing the coefficients of the system (8) in this case, we find:

$$\begin{aligned} \overset{\circ}{\Gamma}_{13} &= -[e_{11} + \mathrm{i}(e_{21} + e_{16}) \frac{\partial}{\partial X_2} - e_{26} \frac{\partial^2}{\partial X_2^2}] \overset{\circ}{E}_3 - [\eta_{11} + 2\mathrm{i}\eta_{12} \frac{\partial}{\partial X_2} - \eta_{22} \frac{\partial^2}{\partial X_2^2}] \overset{\circ}{E}_1 \overset{\circ}{E}_3, \\ \overset{\circ}{\Gamma}_{23} &= -[e_{16} + \mathrm{i}(e_{26} + e_{12}) \frac{\partial}{\partial X_2} - e_{22} \frac{\partial^2}{\partial X_2^2}] \overset{\circ}{E}_3 - [\eta_{11} + 2\mathrm{i}\eta_{12} \frac{\partial}{\partial X_2} - \eta_{22} \frac{\partial^2}{\partial X_2^2}] \overset{\circ}{E}_2 \overset{\circ}{E}_3, \\ \overset{\circ}{\gamma}_3 &= (\eta_{11} + 2\mathrm{i}\eta_{12} \frac{\partial}{\partial X_2} - \eta_{22} \frac{\partial^2}{\partial X_2^2}) \overset{\circ}{E}_3 . \end{aligned} \tag{15}$$

It yields that $\overset{\circ}{\Gamma}_{13} = \overset{\circ}{\Gamma}_{23} = 0$ and $\overset{\circ}{\gamma}_3 = 0$ if $\overset{\circ}{E}_3 = 0$.

Thus, *if the axis* x_3 *is an inverse dyad axis and if* $\overset{\circ}{E}_3 = 0$, *the fundamental system* (8) *splits into two parts*, as follows.

- The first subsystem has the form:

$$\begin{pmatrix} \overset{\circ}{\Gamma}_{11} - \overset{\circ}{\rho} V^2 & \overset{\circ}{\Gamma}_{12} & \overset{\circ}{\gamma}_1 \\ \overset{\circ}{\Gamma}_{12} & \overset{\circ}{\Gamma}_{22} - \overset{\circ}{\rho} V^2 & \overset{\circ}{\gamma}_2 \\ \overset{\circ}{\gamma}_1 & \overset{\circ}{\gamma}_2 & -\overset{\circ}{\epsilon} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_4 \end{pmatrix} = 0. \tag{16}$$

It has as solution a guided wave with sagittal plane polarization, associated with the electric field (*via* the electric potential $u_4 = \varphi$), providing piezoelectric and electrostrictive effects, and depending on the initial stress and electric fields. It is denoted by $\overset{\circ}{\overline{P}}_2$. The electric field, associated with this wave, is contained in the sagittal plane, since $E_3 = \partial\varphi/\partial x_3 = 0$. This fact is consistent with the hypothesis $\overset{\circ}{E}_3 = 0$. These features of $\overset{\circ}{\overline{P}}_2$ wave are obtained from the analysis of the corresponding coefficients:

$$\begin{gathered}
\overset{\circ}{\Gamma}_{11} = c_{11} + \overset{\circ}{S}_{11} - 2e_{11} \overset{\circ}{E}_1 - \eta_{11} \overset{\circ}{E}{}_1^2 + 2\mathrm{i}[c_{16} + \overset{\circ}{S}_{12} - (e_{16} + e_{21}) \overset{\circ}{E}_1 - \eta_{12} \overset{\circ}{E}{}_1^2] \frac{\partial}{\partial X_2} \\
-(c_{66} + \overset{\circ}{S}_{22} - 2e_{26} \overset{\circ}{E}_1 - \eta_{22} \overset{\circ}{E}{}_1^2) \frac{\partial^2}{\partial X_2^2}, \\
\overset{\circ}{\Gamma}_{12} = c_{16} - e_{16} \overset{\circ}{E}_1 - e_{11} \overset{\circ}{E}_2 - \eta_{11} \overset{\circ}{E}_1 \overset{\circ}{E}_2 + \mathrm{i}[c_{12} + c_{66} - (e_{12} + e_{26}) \overset{\circ}{E}_1 \\
-(e_{21} + e_{16}) \overset{\circ}{E}_2 - 2\eta_{12} \overset{\circ}{E}_1 \overset{\circ}{E}_2] \frac{\partial}{\partial X_2} - (c_{26} - e_{22} \overset{\circ}{E}_1 - e_{26} \overset{\circ}{E}_2 - \eta_{22} \overset{\circ}{E}_1 \overset{\circ}{E}_2) \frac{\partial^2}{\partial X_2^2}, \\
\overset{\circ}{\Gamma}_{22} = c_{66} + \overset{\circ}{S}_{11} - 2e_{16} \overset{\circ}{E}_2 - \eta_{11} \overset{\circ}{E}{}_2^2 + 2\mathrm{i}[c_{26} + \overset{\circ}{S}_{12} - (e_{26} + e_{12}) \overset{\circ}{E}_2 - \eta_{12} \overset{\circ}{E}{}_2^2] \frac{\partial}{\partial X_2} \\
-(c_{22} + \overset{\circ}{S}_{22} - 2e_{22} \overset{\circ}{E}_2 - \eta_{22} \overset{\circ}{E}{}_2^2) \frac{\partial^2}{\partial X_2^2},
\end{gathered} \tag{17}$$

respectively:

$$\begin{gathered}
\overset{\circ}{\gamma}_1 = e_{11} + \eta_{11} \overset{\circ}{E}_1 + \mathrm{i}(e_{16} + e_{21} + 2\eta_{12} \overset{\circ}{E}_1) \frac{\partial}{\partial X_2} - (e_{26} + \eta_{22} \overset{\circ}{E}_1) \frac{\partial^2}{\partial X_2^2}, \\
\overset{\circ}{\gamma}_2 = e_{16} + \eta_{11} \overset{\circ}{E}_2 + \mathrm{i}(e_{12} + e_{26} + 2\eta_{12} \overset{\circ}{E}_2) \frac{\partial}{\partial X_2} - (e_{22} + \eta_{22} \overset{\circ}{E}_2) \frac{\partial^2}{\partial X_2^2}, \\
\overset{\circ}{\epsilon} = \overset{\circ}{\epsilon}_{11} + 2\mathrm{i} \overset{\circ}{\epsilon}_{12} \frac{\partial}{\partial X_2} - \overset{\circ}{\epsilon}_{22} \frac{\partial^2}{\partial X_2^2} = 1 + \eta_{11} + 2\mathrm{i}\eta_{12} \frac{\partial}{\partial X_2} - (1 + \eta_{22}) \frac{\partial^2}{\partial X_2^2}.
\end{gathered} \tag{18}$$

- The second subsystem reduces to a single equation, as follows:

$$(\overset{\circ}{\Gamma}_{33} - \overset{\circ}{\rho} V^2) u_3 = 0. \tag{19}$$

Its root corresponds to a transverse-horizontal wave, non-piezoelectric, and influenced by the initial stress field, only. It is called $\overset{\circ}{T}H$ wave. In this equation:

$$\overset{\circ}{\Gamma}_{33}= c_{55}+ \overset{\circ}{S}_{11} +2\mathrm{i}(c_{45}+ \overset{\circ}{S}_{12})\frac{\partial}{\partial X_2} - (c_{44}+ \overset{\circ}{S}_{22})\frac{\partial^2}{\partial X_2^2}. \tag{20}$$

In conclusion, in this paper we studied the coupling conditions for propagation of planar guided waves in a piezoelectric semi-infinite plane (i.e. sagittal plane). If the sagittal plane is normal to a direct, resp. inverse dyad axis, we derive that the fundamental equations' system decomposes for particular choices of the initial electric field. In this way we obtain mechanical and piezoelectric waves generalizing the classical guided waves from the case without initial fields. These results will help us to determine the corresponding boundary conditions and to derive the guided waves velocities.

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