A NEW APPROACH FOR SOLITON SOLUTIONS OF RLW EQUATION AND (1+2)-DIMENSIONAL NONLINEAR SCHRÖDINGER'S EQUATION

ALI FILIZ, ABDULLAH SONMEZOGLU, MEHMET EKICI and DURGUN DURAN

Communicated by Horia Cornean

In this paper, we introduce a new version of the trial equation method for solving non-integrable partial differential equations in mathematical physics. The exact traveling wave solutions including soliton solutions, singular soliton solutions, rational function solutions and elliptic function solutions to the RLW equation and (1+2)-dimensional nonlinear Schrödinger's equation in dual-power law media are obtained by this method.

AMS 2010 Subject Classification: 35Q51, 47J35, 74J35.

Key words: extended trial equation method, RLW equation, (1+2)-dimensional nonlinear Schrödinger's equation in dual-power law media, soliton solution, elliptic solutions.

1. INTRODUCTION

In recent years, there have been important and far reaching developments in the study of nonlinear waves and a class of nonlinear wave equations which arise frequently in applications. The wide interest in this field comes from the understanding of special waves called solitons and the associated development of a method of solution to two class of nonlinear wave equations termed the regularized long-wave (RLW) equation and the nonlinear Schrödinger's equation (NLSE). The RLW equation arises in the study of shallow-water waves. The generalized version of the RLW equation is known as the $R(m,n)$ equation. The NLSE is an example of a universal nonlinear model that describes many physical nonlinear systems. The equation can be applied to hydrodynamics, nonlinear optics, nonlinear acoustics, quantum condensates, heat pulses in solids and various other nonlinear instability phenomena. A soliton phenomenon is an attractive field of present day research in nonlinear physics and mathematics. Essential ingredients in the soliton theory are the RLW equation and the NLSE, and their variants appearing in a wide spectrum of problems. Solitons are identified with a certain class of reflectionless solutions of the integrable equations. Such equations, including the RLW equation and the NLSE, are named soliton equations. More exactly, solitons are identified with a certain class of reflectionless solutions of the integrable equations. At every instant a soliton is localized in a restricted spatial region with its centroid moving like a particle. The particle-like properties of solitons are also manifested in their elastic collisions.

Soliton equations make up a narrow class of nonlinear equations, whereas a wider set of nonlinear equations, being non-integrable, possess soliton-like solutions. They are localized in some sense, propagate with small energy losses, and collide with a varied extent of inelasticity. These solutions are termed solitary waves, quasisolitons, soliton-like solutions, etc. to differentiate them from the solitons in the above exact meaning. The stability of the localized form of solitons and solitary waves and their elastic collisions have led to interesting physical applications.

Constructing exact solutions to partial differential equations is an important problem in nonlinear science. In order to obtain the exact solutions of nonlinear partial differential equations, various methods have been presented, such as tanh-coth method [1, 15], Hirota method [9], the exponential function method [6, 17], (G'/G)-expansion method [7], the trial equation method [4, 5, 8, 10–14, 16] and so on. There are a lot of nonlinear evolution equations that are integrated using these and other mathematical methods. Soliton solutions, compactons, peakons, cuspons, stumpons, cnoidal waves, singular solitons and other solutions have been found. These types of solutions are very important and appear in various areas of physics, applied mathematics.

In the next section, we give a new version of the trial equation method for nonlinear differential equations with generalized evolution. We will present some exact solutions to two nonlinear problems with higher nonlinear terms such as the RLW equation [2]

$$u_t + \alpha u_x + \beta u^m u_x + \gamma u_{xxt} = 0, \tag{1.1}$$

and the (1+2)-dimensional NLSE in dual-power law media [3]

$$iq_t + \frac{1}{2}(q_{xx} + q_{yy}) + \left(|q|^{2m} + k|q|^{4m}\right) q = 0, \tag{1.2}$$

in the framework of a new approximation of the trial equation method for nonlinear waves and analyze some of their remarkable features. In physics, a wave is a disturbance (an oscillation) that travels through space and time, accompanied by the transfer of energy. Travelling wave is a function u of the form

$$u(x,t) = f(x - ct),$$

where $f : R \to V$ is a function defining the wave shape, and c is a real number defining the propagation speed of the wave.

2. THE EXTENDED TRIAL EQUATION METHOD

The main steps of the extended trial equation method for the nonlinear partial differential equation with **rank inhomogeneous** [12] are outlined as follows.

Step 1. Consider a nonlinear partial differential equation

$$P(u, u_t, u_x, u_{xx}, \dots) = 0, \tag{2.3}$$

where P is a polynomial. Take a wave transformation as

$$u(x_1, \dots, x_N, t) = u(\eta), \quad \eta = \lambda\left(\sum_{j=1}^{N} x_j - ct\right), \tag{2.4}$$

where $\lambda \neq 0$ and $c \neq 0$. Substituting Eq. (2.4) into Eq. (2.3) yields a nonlinear ordinary differential equation,

$$N(u, u', u'', ...) = 0. \tag{2.5}$$

Step 2. Take transformation and trial equation as follows:

$$u = \sum_{i=0}^{\delta} \tau_i \Gamma^i, \tag{2.6}$$

in which

$$(\Gamma')^2 = \Lambda(\Gamma) = \frac{\Phi(\Gamma)}{\Psi(\Gamma)} = \frac{\xi_\theta \Gamma^\theta + ... + \xi_1 \Gamma + \xi_0}{\zeta_\epsilon \Gamma^\epsilon + ... + \zeta_1 \Gamma + \zeta_0}, \tag{2.7}$$

where $\tau_i\,(i = 0, ..., \delta)$, $\xi_i\,(i = 0, ..., \theta)$ and $\zeta_i\,(i = 0, ..., \epsilon)$ are constants. Using the relations (2.6) and (2.7), we can find

$$(u')^2 = \frac{\Phi(\Gamma)}{\Psi(\Gamma)}\left(\sum_{i=0}^{\delta} i\tau_i \Gamma^{i-1}\right)^2, \tag{2.8}$$

(2.9)

$$u'' = \frac{\Phi'(\Gamma)\Psi(\Gamma) - \Phi(\Gamma)\Psi'(\Gamma)}{2\Psi^2(\Gamma)}\left(\sum_{i=0}^{\delta} i\tau_i \Gamma^{i-1}\right) + \frac{\Phi(\Gamma)}{\Psi(\Gamma)}\left(\sum_{i=0}^{\delta} i(i-1)\tau_i \Gamma^{i-2}\right),$$

where $\Phi(\Gamma)$ and $\Psi(\Gamma)$ are polynomials. Substituting these terms into Eq. (2.5) yields an equation of polynomial $\Omega(\Gamma)$ of Γ :

$$\Omega(\Gamma) = \varrho_s \Gamma^s + ... + \varrho_1 \Gamma + \varrho_0 = 0. \tag{2.10}$$

According to the balance principle we can determine a relation of θ, ϵ, and δ. We can take some values of θ, ϵ, and δ.

Step 3. Let the coefficients of $\Omega(\Gamma)$ all be zero will yield an algebraic equations system:

$$\varrho_i = 0, \quad i = 0, ..., s. \tag{2.11}$$

Solving this equations system (2.11), we will determine the values of $\xi_0, ..., \xi_\theta$; $\zeta_0, ..., \zeta_\epsilon$ and $\tau_0, ..., \tau_\delta$.

Step 4. Reduce Eq. (2.7) to the elementary integral form,

$$\pm(\eta - \eta_0) = \int \frac{\mathrm{d}\Gamma}{\sqrt{\Lambda(\Gamma)}} = \int \sqrt{\frac{\Psi(\Gamma)}{\Phi(\Gamma)}}\mathrm{d}\Gamma. \tag{2.12}$$

Using a complete discrimination system for polynomial to classify the roots of $\Phi(\Gamma)$, we solve the infinite integral (2.12) and obtain the exact solutions to Eq. (2.5). Furthermore, we can write the exact traveling wave solutions to Eq. (2.3) respectively.

3. APPLICATIONS

To illustrate the necessity of our new approach concerning the trial equation method, we introduce two case studies.

Example 3.1 (Application to the RLW equation). In Eq. (1.1), α, β and γ are free parameters, and the parameter m dictates the power-law nonlinearity. The first term is the evolution term, and the third term is the nonlinear term, while the second and fourth terms are the dispersion terms. The solitons are the result of a delicate balance between dispersion and nonlinearity.

In order to look for travelling wave solutions of Eq. (1.1), we make the transformation

$$u(x,t) = u(\eta), \quad \eta = x - ct,$$

where c is an arbitrary constant. Then, integrating this equation and setting the integration constant to zero, we obtain

$$(\alpha - c)u + \frac{\beta}{m+1}u^{m+1} - c\gamma u'' = 0, \tag{3.13}$$

where m is a positive integer. Eq. (3.13), with the transformation

$$u = v^{1/m}, \tag{3.14}$$

reduces to

$$cMvv'' + cN(v')^2 + (c-\alpha)v^2 - Pv^3 = 0, \tag{3.15}$$

where $M=\gamma/m$, $N=\gamma(1-m)/m^2$, $P=\beta/(m+1)$. Substituting Eqs. (2.8) and (2.9) into Eq. (3.15) and using balance principle yields $\theta=\epsilon+\delta+2$. If we take $\theta=3$, $\epsilon=0$ and $\delta=1$, then

$$(v')^2=\frac{\tau_1^2(\xi_3\Gamma^3+\xi_2\Gamma^2+\xi_1\Gamma+\xi_0)}{\zeta_0},$$

where $\xi_3\neq 0$, $\zeta_0\neq 0$. Respectively, solving the algebraic equation system (2.11) yields

$$\begin{aligned}
\xi_0 &= \frac{\tau_0^2(-4\zeta_0\tau_0P+2\xi_2\tau_0P(M+N)+\alpha\xi_2(3M+2N))}{\tau_1^2(6\tau_0P(M+N)+\alpha(3M+2N))},\\
\xi_1 &= \frac{2\tau_0(-3\zeta_0\tau_0P+3\xi_2\tau_0P(M+N)+\alpha\xi_2(3M+2N))}{\tau_1(6\tau_0P(M+N)+\alpha(3M+2N))},\\
\xi_3 &= \frac{2\tau_1P(\zeta_0+\xi_2(M+N))}{6\tau_0P(M+N)+\alpha(3M+2N)},\\
c &= \frac{\zeta_0(6\tau_0P(M+N)+\alpha(3M+2N))}{(3M+2N)(\zeta_0+\xi_2(M+N))},\\
\xi_2 &= \xi_2,\quad \zeta_0=\zeta_0,\quad \tau_0=\tau_0,\quad \tau_1=\tau_1.
\end{aligned}$$

Substituting these results into Eq. (2.7) and Eq. (2.12), we can write (3.16)

$$\pm(\eta-\eta_0)=\sqrt{\frac{\zeta_0(6\tau_0P(M+N)+\alpha(3M+2N))}{2\tau_1P(\zeta_0+\xi_2(M+N))}}\times\int\frac{d\Gamma}{\sqrt{\Gamma^3+\ell_2\Gamma^2+\ell_1\Gamma+\ell_0}},$$

where

$$\begin{aligned}
\ell_2 &= \frac{\xi_2(6\tau_0P(M+N)+\alpha(3M+2N))}{2\tau_1P(\zeta_0+\xi_2(M+N))},\\
\ell_1 &= \frac{\tau_0(-3\zeta_0\tau_0P+3\xi_2\tau_0P(M+N)+\alpha\xi_2(3M+2N))}{\tau_1^2P(\zeta_0+\xi_2(M+N))},\\
\ell_0 &= \frac{\tau_0^2(-4\zeta_0\tau_0P+2\xi_2\tau_0P(M+N)+\alpha\xi_2(3M+2N))}{2\tau_1^3P(\zeta_0+\xi_2(M+N))}.
\end{aligned}$$

Integrating Eq. (3.16), we obtain the solutions to the Eq. (1.1) as follows:

$$\pm(\eta-\eta_0)=-2\sqrt{A}\frac{1}{\sqrt{\Gamma-\alpha_1}}, \tag{3.17}$$

$$\pm(\eta-\eta_0)=2\sqrt{\frac{A}{\alpha_2-\alpha_1}}\arctan\sqrt{\frac{\Gamma-\alpha_2}{\alpha_2-\alpha_1}},\quad \alpha_2>\alpha_1, \tag{3.18}$$

$$\pm(\eta-\eta_0)=\sqrt{\frac{A}{\alpha_1-\alpha_2}}\ln\left|\frac{\sqrt{\Gamma-\alpha_2}-\sqrt{\alpha_1-\alpha_2}}{\sqrt{\Gamma-\alpha_2}+\sqrt{\alpha_1-\alpha_2}}\right|,\quad \alpha_1>\alpha_2, \tag{3.19}$$

$$\pm(\eta-\eta_0)=2\sqrt{\frac{A}{\alpha_1-\alpha_3}}F(\varphi,l),\quad \alpha_1>\alpha_2>\alpha_3, \tag{3.20}$$

where

$$A=\frac{\zeta_0(6\tau_0P(M+N)+\alpha(3M+2N))}{2\tau_1P(\zeta_0+\xi_2(M+N))},\quad F(\varphi,l)=\int_0^{\varphi}\frac{\mathrm{d}\psi}{\sqrt{1-l^2\sin^2\psi}},$$

and

$$\varphi=\arcsin\sqrt{\frac{\Gamma-\alpha_3}{\alpha_2-\alpha_3}},\quad l^2=\frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3}.$$

Also α_1, α_2 and α_3 are the roots of the polynomial equation

$$\Gamma^3+\frac{\xi_2}{\xi_3}\Gamma^2+\frac{\xi_1}{\xi_3}\Gamma+\frac{\xi_0}{\xi_3}=0.$$

Substituting the solutions (3.17)–(3.19) into (2.6) and (3.14), denoting $\bar{\tau}=\tau_0+\tau_1\alpha_1$, and setting

$$v=\frac{\zeta_0(6\tau_0P(M+N)+\alpha(3M+2N))}{(3M+2N)(\zeta_0+\xi_2(M+N))},$$

we get

$$u(x,t)=\left[\bar{\tau}+\frac{4\tau_1A}{(x-vt-\eta_0)^2}\right]^{\frac{1}{m}}, \tag{3.21}$$

(3.22)

$$u(x,t)=\left\{\bar{\tau}+\tau_1(\alpha_2-\alpha_1)\left[1-\tanh^2\left(\mp\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A}}(x-vt-\eta_0)\right)\right]\right\}^{\frac{1}{m}},$$

$$u(x,t)=\left\{\bar{\tau}+\tau_1(\alpha_1-\alpha_2)\mathrm{cosech}^2\left(\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A}}(x-vt)\right)\right\}^{\frac{1}{m}}. \tag{3.23}$$

If we take $\tau_0=-\tau_1\alpha_1$, that is $\bar{\tau}=0$, and $\eta_0=0$, then the solutions (3.21)–(3.23) can reduce to rational function solution

$$u(x,t)=\left[\frac{2\sqrt{\tau_1A}}{x-vt}\right]^{\frac{2}{m}}, \tag{3.24}$$

1-soliton solution (see Figure 1)

$$u(x,t)=\frac{A_1}{\cosh^{\frac{2}{m}}[\mp B(x-vt)]}, \tag{3.25}$$

and singular soliton solution (see Figure 2)

$$u(x,t)=\frac{A_2}{\sinh^{\frac{2}{m}}[B(x-vt)]}, \tag{3.26}$$

where

$$A_1 = [\tau_1(\alpha_2 - \alpha_1)]^{\frac{1}{m}}, \quad A_2 = [\tau_1(\alpha_1 - \alpha_2)]^{\frac{1}{m}}, \quad B = \frac{1}{2}\sqrt{\frac{\alpha_1 - \alpha_2}{A}}.$$

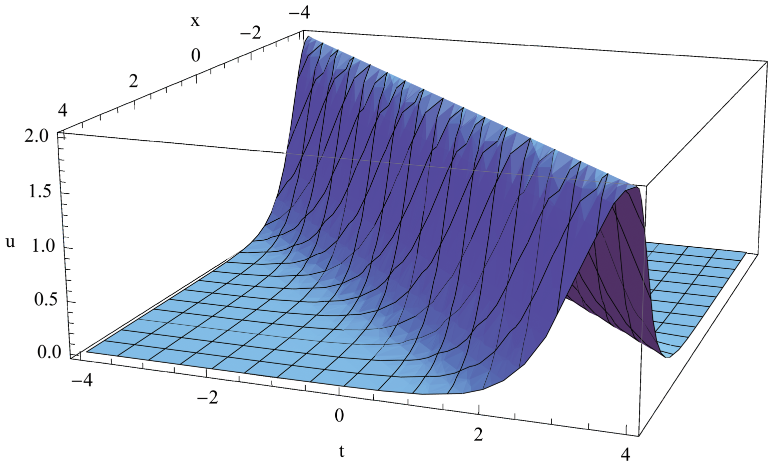

Fig. 1 – Profile of the solution (3.25) corresponding to the values $A_1 = 2,\ B = 1,\ m = 1$ and $v = 1$.

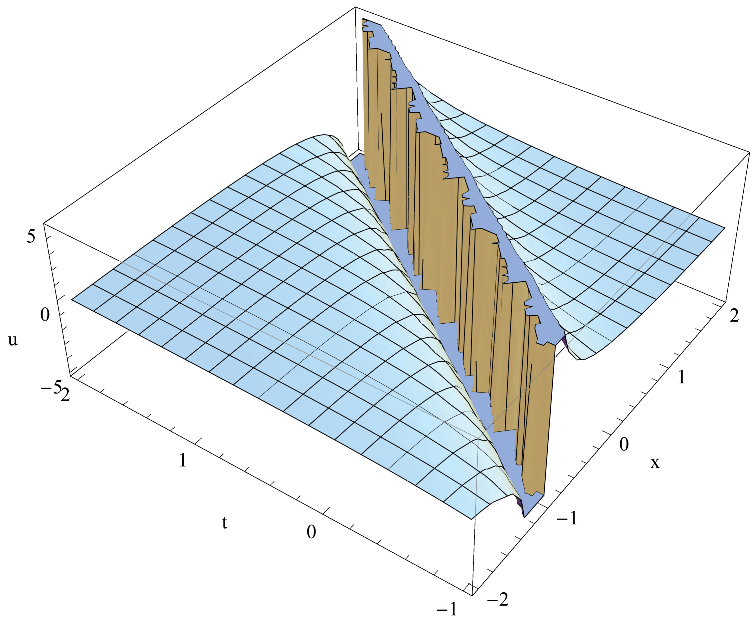

Fig. 2 – Profile of the solution (3.26) corresponding to the values $A_2 = 2,\ B = 1,\ m = 2$ and $v = 1$.

Here, A_1 and A_2 are the amplitudes of the solitons, while v is the velocity and B is the inverse width of the solitons. Thus, we can say that the solitons exist for $\tau_1 > 0$.

Remark 3.2. If we let the corresponding values for some parameters, solution (3.25) is respectively in full agree with the solution (13) and the solution (33) mentioned in Refs. [2, 4].

Remark 3.3. The solutions (3.24)–(3.26) obtained by using the extended trial equation method for Eq. (1.1) have been checked using one of the symbolic computation programming in Mathematica. To our knowledge, the rational function solution, 1-soliton solution and the singular soliton solution, that we find in this paper, are not shown in the previous literature. These results are new exact solutions of Eq. (1.1).

Example 3.4 (Application to (1+2)-dimensional nonlinear Schrödinger's equation in dual-power law media). In Eq. (1.2), the first term represents the evolution term, the second and third terms, in parenthesis, represent the dispersion in x and y directions while the fourth and fifth terms in parenthesis together represents nonlinearity, where k is a constant. Solitons are the result of a delicate balance between dispersion and nonlinearity.

In order to look for travelling wave solutions of Eq. (1.2), we make the transformation

$$q(x,y,t) = u(\eta)e^{i\phi}, \tag{3.27}$$

$$\eta = B_1 x + B_2 y - ct, \quad \phi = -\kappa_1 x - \kappa_2 y + \omega t + \varsigma, \tag{3.28}$$

where $B_1, B_2, c, \kappa_1, \kappa_2, \omega$ and ς are real constants. Then, substituting relations (3.27) and (3.28) into (1.2) and decomposing into real and imaginary parts, respectively, yields

$$-(2\omega + \kappa_1^2 + \kappa_2^2)u + 2u^{2m+1} + 2ku^{4m+1} + (B_1^2 + B_2^2)u'' = 0, \tag{3.29}$$

and

$$-2(c + \kappa_1 B_1 + \kappa_2 B_2)u' = 0, \tag{3.30}$$

where m is a positive integer. Eq. (3.29), with the transformation

$$u = v^{1/2m}, \tag{3.31}$$

reduces to

$$Mvv'' + N(v')^2 - Pv^2 + 2v^3 + 2kv^4 = 0, \tag{3.32}$$

where $M = \left(B_1^2 + B_2^2\right)/2m, \quad N = (1-2m)\left(B_1^2 + B_2^2\right)/4m^2, \quad P = 2\omega + \kappa_1^2 + \kappa_2^2$. Substituting Eqs. (2.8) and (2.9) into Eq. (3.32) and using balance principle yields $\theta = \epsilon + 2\delta + 2$. If we take $\theta = 4$, $\epsilon = 0$ and $\delta = 1$, then

$$(v')^2 = \frac{\tau_1^2(\xi_4\Gamma^4 + \xi_3\Gamma^3 + \xi_2\Gamma^2 + \xi_1\Gamma + \xi_0)}{\zeta_0},$$

where $\xi_4 \neq 0$, $\zeta_0 \neq 0$. Respectively, solving the algebraic equation system (2.11) yields

$$\begin{aligned}
\xi_0 &= \frac{\xi_3\tau_0^2(2\tau_0(M+N)(M(4+3k\tau_0)+2N(1+k\tau_0))-Q)}{4\tau_1^3(M+N)(2M(1+3k\tau_0)+N+4k\tau_0N)},\\
\xi_1 &= \frac{\xi_3\tau_0(2\tau_0(M+N)(6M(1+k\tau_0)+N(3+4k\tau_0))-Q)}{2\tau_1^2(M+N)(2M(1+3k\tau_0)+N+4k\tau_0N)},\\
\xi_2 &= \frac{12\xi_3\tau_0(M+N)(M(2+3k\tau_0)+N+2k\tau_0N)-\xi_3Q}{4\tau_1(M+N)(2M(1+3k\tau_0)+N+4k\tau_0N)},\\
\xi_3 &= \xi_3, \quad \xi_4 = \frac{k\xi_3\tau_1(3M+2N)}{2(2M(1+3k\tau_0)+N+4k\tau_0N)},\\
\zeta_0 &= -\frac{\xi_3Q}{4\tau_1P(2M(1+3k\tau_0)+N+4k\tau_0N)},\\
\tau_0 &= \tau_0, \quad \tau_1 = \tau_1,
\end{aligned}$$

where $Q = P(2M+N)(3M+2N)$. Also from Eq. (3.30), it can be seen that $c = -(\kappa_1B_1 + \kappa_2B_2)$. Substituting these results into Eq. (2.7) and Eq. (2.12), we can write

$$\pm(\eta - \eta_0) = \sqrt{-\frac{2M+N}{2k\tau_1^2}} \times \int \frac{\mathrm{d}\Gamma}{\sqrt{\Gamma^4 + \ell_3\Gamma^3 + \ell_2\Gamma^2 + \ell_1\Gamma + \ell_0}}, \tag{3.33}$$

where

$$\begin{aligned}
\ell_3 &= \frac{4M(1+3k\tau_0)+2N(1+4k\tau_0)}{k\tau_1(3M+2N)},\\
\ell_2 &= \frac{1}{2k\tau_1^2}\left[6\tau_0\left(1+2k\tau_0+\frac{M}{3M+2N}\right)-\frac{P(2M+N)}{M+N}\right],\\
\ell_1 &= \frac{\tau_0}{k\tau_1^3}\left[\tau_0\left(3+4k\tau_0+\frac{3M}{3M+2N}\right)-\frac{P(2M+N)}{M+N}\right],\\
\ell_0 &= \frac{\tau_0^2}{2k\tau_1^4}\left[2\tau_0\left(1+k\tau_0+\frac{M}{3M+2N}\right)-\frac{P(2M+N)}{M+N}\right].
\end{aligned}$$

Integrating Eq. (3.33), we obtain the solutions to the Eq. (1.2) as follows:

$$\pm(\eta - \eta_0) = -\frac{B}{\Gamma - \alpha_1}, \tag{3.34}$$

(3.35) $$\pm(\eta-\eta_0)=\frac{2B}{\alpha_1-\alpha_2}\sqrt{\frac{\Gamma-\alpha_2}{\Gamma-\alpha_1}},\quad \alpha_2>\alpha_1,$$

(3.36) $$\pm(\eta-\eta_0)=\frac{B}{\alpha_1-\alpha_2}\ln\left|\frac{\Gamma-\alpha_1}{\Gamma-\alpha_2}\right|,$$

(3.37) $$\begin{aligned}\pm(\eta-\eta_0) &= \frac{B}{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}\\ &\quad\times\ln\left|\frac{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}-\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}+\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}\right|,\\ &\quad\alpha_1>\alpha_2>\alpha_3,\end{aligned}$$

(3.38) $$\pm(\eta-\eta_0)=2\sqrt{\frac{B}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}F(\varphi,l),\quad \alpha_1>\alpha_2>\alpha_3>\alpha_4,$$

where

$$B=\sqrt{-\frac{2M+N}{2k\tau_1^2}},\quad F(\varphi,l)=\int_0^{\varphi}\frac{d\psi}{\sqrt{1-l^2\sin^2\psi}},$$

and

$$\varphi=\arcsin\sqrt{\frac{(\Gamma-\alpha_1)(\alpha_2-\alpha_4)}{(\Gamma-\alpha_2)(\alpha_1-\alpha_4)}},\quad l^2=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}.$$

Also α_1, α_2, α_3 and α_4 are the roots of the polynomial equation

$$\Gamma^4+\frac{\xi_3}{\xi_4}\Gamma^3+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_0}{\xi_4}=0.$$

Substituting the solutions (3.34)–(3.37) into (2.6) and (3.31), and setting $v=-(\kappa_1B_1+\kappa_2B_2)$, we obtain

(3.39) $$q(x,y,t)=\left\{\tau_0+\tau_1\alpha_1\mp\frac{\tau_1B}{B_1x+B_2y-vt-\eta_0}\right\}^{\frac{1}{2m}}e^{i\phi},$$

(3.40)
$$q(x,y,t)=\left\{\tau_0+\tau_1\alpha_1+\frac{4B^2(\alpha_2-\alpha_1)\tau_1}{4B^2-\left[(\alpha_1-\alpha_2)\left(B_1x+B_2y-vt-\eta_0\right)\right]^2}\right\}^{\frac{1}{2m}}e^{i\phi},$$

(3.41)
$$q(x,y,t)=\left\{\tau_0+\tau_1\alpha_2+\frac{(\alpha_2-\alpha_1)\tau_1}{\exp\left(\frac{\alpha_1-\alpha_2}{B}\left(B_1x+B_2y-vt-\eta_0\right)\right)-1}\right\}^{\frac{1}{2m}}e^{i\phi},$$

(3.42)

$$q(x,y,t)=\left\{\tau_0+\tau_1\alpha_1+\frac{(\alpha_1-\alpha_2)\tau_1}{\exp\left(\frac{\alpha_1-\alpha_2}{B}\left(B_1x+B_2y-vt-\eta_0\right)\right)-1}\right\}^{\frac{1}{2m}}e^{i\phi},$$

(3.43)

$$q(x,y,t)=\left\{\tau_0+\tau_1\alpha_1-\frac{2\hat{A}_{22}\tau_1}{\hat{A}_{11}+(\alpha_3-\alpha_2)\cosh\left(\frac{\sqrt{\hat{A}_{22}}}{B}\left(\hat{A}_{33}\right)\right)}\right\}^{\frac{1}{2m}}e^{i\phi},$$

where $\hat{A}_{11}=2\alpha_1-\alpha_2-\alpha_3$, $\hat{A}_{22}=(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)$ and $\hat{A}_{33}=B_1x+B_2y-vt$. If we take $\tau_0=-\tau_1\alpha_1$ and $\eta_0=0$, then the solutions (3.39)–(3.43) can reduce to rational function solutions

$$q(x,y,t)=\left(\mp\frac{\tau_1 B}{B_1x+B_2y-vt}\right)^{\frac{1}{2m}}e^{i(-\kappa_1x-\kappa_2y+\omega t+\varsigma)}, \quad (3.44)$$

(3.45)

$$q(x,y,t)=\left\{\frac{4B^2(\alpha_2-\alpha_1)\tau_1}{4B^2-\left[(\alpha_1-\alpha_2)\left(B_1x+B_2y-vt\right)\right]^2}\right\}^{\frac{1}{2m}}e^{i(-\kappa_1x-\kappa_2y+\omega t+\varsigma)},$$

traveling wave solutions

(3.46)

$$q(x,y,t)=\left\{\frac{(\alpha_2-\alpha_1)\tau_1}{2}\left\{1\mp\coth\left[\frac{\alpha_1-\alpha_2}{2B}\left(B_1x+B_2y-vt\right)\right]\right\}\right\}^{\frac{1}{2m}}e^{i(-\kappa_1x-\kappa_2y+\omega t+\varsigma)},$$

and soliton solution (see Figure 3)

(3.47)

$$q(x,y,t)=\frac{A_3}{\left(D+\cosh\left[K(B_1x+B_2y-vt)\right]\right)^{\frac{1}{2m}}}e^{i(-\kappa_1x-\kappa_2y+\omega t+\varsigma)},$$

where

$$A_3=\left(\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\tau_1}{\alpha_3-\alpha_2}\right)^{\frac{1}{2m}},\ D=\frac{2\alpha_1-\alpha_2-\alpha_3}{\alpha_3-\alpha_2},\ K=\frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{B}.$$

Here in (3.47), A_3 is the amplitude of the soliton, B_1 is the inverse width of soliton in the $x-$direction and B_2 is the inverse width of soliton in the

$y-$direction and v is the velocity of the soliton. Also κ_1 and κ_2 represents the soliton frequency in the x and y directions respectively, while ω represents the solitary wave number and finally ς is the phase constant of the soliton. Thus, we can say that the solitons exist for $\tau_1 < 0$.

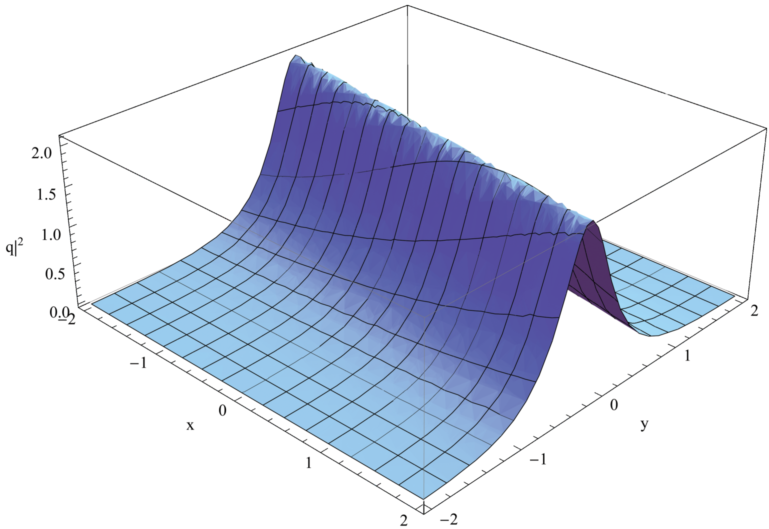

Fig. 3 – Profile of the solution (3.47) corresponding to the values $A_3^2 = 8/3,\ B = 1,\ B_1 = 1,\ B_2 = 6,\ D = 5/3,\ K = 2,\ m = 1$ while $vt = 1$.

Remark 3.5. If we let the corresponding values for some parameters, solution (3.47) is in full agree with the solution (16) mentioned in Ref. [3].

Remark 3.6. The solutions (3.44)–(3.47) obtained by using the extended trial equation method for Eq. (1.2) have been checked again by Mathematica. To our knowledge, the rational function solutions and the soliton solution obtained using the method described in this paper are not shown in the previous literature. These are new wave solutions of Eq. (1.2).

4. **CONCLUSION**

In this brief review we introduced the RLW equation and the NLSE, and discussed some of their remarkable features. The 1-soliton solution separates between periodic wave solutions. Basic features of the 1-soliton solution, the singular soliton solution and the soliton solution were discussed. We proposed a new trial equation method as an alternative approach to obtain the analytic solutions of nonlinear partial differential equations with generalized evolution in mathematical physics. In the quadratic choice of the trial equation, the function N of (2.5) is a polynomial where only even powers of $(u')^2$ appear.

Consequently, the solutions of the extended trial equation method includes the solutions obtained by the standard approach [12]. One obtains only the exact traveling wave solutions by the standart method. On the other hand, with the extended trial equation method, we can get not only the exact traveling wave solutions, but the soliton solutions, the singular soliton solutions, the rational function solutions and the elliptic function solutions, as well. We use the extended trial equation method aided with symbolic computation to construct the soliton solutions, the elliptic function and rational function solutions for the RLW equation and (1+2)-dimensional NLSE in dual-power law media. The elliptic function solutions obtained by the present approach are more general than those obtained earlier.

Acknowledgments. We would like to thank the referees for their valuable suggestions which improved the manuscript.

REFERENCES

[1] M.A. Abdou, *The extended tanh method and its applications for solving nonlinear physical models*. Appl. Math. Comput. **190** (2007), 988–996.

[2] A. Biswas, *Solitary waves for power-law regularized long-wave equation and R(m,n) equation*. Nonlinear Dyn. **59** (2010), 423–426.

[3] A. Biswas, *1-soliton solution of (1+2)-dimensional nonlinear Schrödinger's equation in dual-power law media*. Phys. Lett. A **372** (2008), 5941–5943.

[4] Y. Gurefe, E. Misirli, A. Sonmezoglu and M. Ekici, *Extended trial equation method to generalized nonlinear partial differential equations*. Appl. Math. Comput. **219** (2013), 5253–5260.

[5] Y. Gurefe, A. Sonmezoglu and E. Misirli, *Application of an irrational trial equation method to high-dimensional nonlinear evolution equations*. J. Adv. Math. Stud. **5**(1) (2012), 41–47.

[6] Y. Gurefe and E. Misirli, *Exp-function method for solving nonlinear evolution equations with higher order nonlinearity*. Comput. Math. Appl. **61** (2011), 2025–2030.

[7] Y. Gurefe and E. Misirli, *New variable separation solutions of two-dimensional Burgers system*. Appl. Math. Comput. **217** (2011), 9189–9197.

[8] Y. Gurefe, A. Sonmezoglu and E. Misirli, *Application of the trial equation method for solving some nonlinear evolution equations arising in mathematical physics*. Pramana-J. Phys. **77**(6) (2011), 1023–1029.

[9] R. Hirota, *Exact solutions of the Korteweg-de-Vries equation for multiple collisions of solitons*. Phys. Lett. A **27** (1971), 1192–1194.

[10] C.S. Liu, *Applications of complete discrimination system for polynomial for classifications of traveling wave solutions to nonlinear differential equations*. Comput. Phys. Commun. **181** (2010), 317–324.

[11] C.S. Liu, *A new trial equation method and its applications*. Commun. Theor. Phys. **45** (2006), 395–397.

[12] C.S. Liu, *Trial equation method for nonlinear evolution equations with rank inhomogeneous: mathematical discussions and applications*. Commun. Theor. Phys. **45** (2006), 219–223.

[13] C.S. Liu, *Trial equation method and its applications to nonlinear evolution equations*. Acta. Phys. Sin. **54** (2005), 2505–2509.

[14] C.S. Liu, *Using trial equation method to solve the exact solutions for two kinds of KdV equations with variable coefficients.* Acta. Phys. Sin. **54** (2005), 4506–4510.

[15] W. Malfliet and W. Hereman, *The tanh method: exact solutions of nonlinear evolution and wave equations.* Phys. Scr. **54** (1996), 563–568.

[16] Y. Pandir, Y. Gurefe, U. Kadak and E. Misirli, *Classification of exact solutions for some nonlinear partial differential equations with generalized evolution.* Abstr. Appl. Anal. **2012** (2012), Article ID 478531, 16 pp.

[17] H.X. Wu and J.H. He, *Exp-function method and its application to nonlinear equations.* Chaos Solitons Fractals **30** (2006), 700–708.

Received 9 December 2012

"Adnan Menderes" University,
Faculty of Science and Arts,
Department of Mathematics,
09010 Aydin, Turkey
afiliz@adu.edu.tr

Bozok University,
Faculty of Science and Arts,
Department of Mathematics,
66100 Yozgat, Turkey
abdullah.sonmezoglu@bozok.edu.tr

Bozok University,
Faculty of Science and Arts,
Department of Mathematics,
66100 Yozgat, Turkey
mehmet.ekici@bozok.edu.tr

Bozok University,
Faculty of Science and Arts,
Department of Physics,
66100 Yozgat, Turkey
durgun.duran@bozok.edu.tr